A possible explanation of the OPERA result based on known GPS Physics

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Abstract

We propose an explanation of OPERA result in terms of a known general relativistic GPS physics, a term called the eccentricity correction. We review the origin and interpretation of such a term in General Relativity. Further comments and suggestions in order to test our proposal are given. We claim that we have not exhausted yet every possible explanation of OPERA data, and then, before considering new physics models that are in tension with other observations, we should try to understand what current theories say. New physics is unlikely the right response, but we do not exclude it if new experiments and data analyses are done and the anomaly persists. However, we should firstly agree if every possible explanation with well established theories are in tension with data and if it has been precisely done. The ultimate fate of this proposal will be elucidated via a forthcoming MINOS experiment and a remake of the CERN to Gran Sasso experiment using an alternative syncronization method based upon optical fiber instead the GPS clock device.

1 Introduction: OPERA results

The CERN to Gran Sasso experiment is a tau neutrino appearing experiment. In addition to this goal, OPERA experiment has tried to measure the velocity of neutrinos that are generated at CERN and received later at the Laboratorio Nazionale del Gran Sasso (LNGS). The method requires accurate knowledge of the distance baseline traveled by the neutrino along with detailed account of any delays in the electronic transmission of timing signals at both ends of the baseline, carried by a GPS system. The overall time of flight is then estimated by time marking local reference clocks at both sites using a single, common view GPS clock signal received separately at both places. The time difference between time ruled clock signals at either end is calibrated using a second atomic clock transported between the two GPS receptors. The stated result, after allowing for all known and alleged cumulated time delays, is that the Time Of Flight (TOF) of the neutrinos is *shorter* by $60 \pm 6.9(stat.) \pm 7.4(sys.)$ns than that expected if they traveled at the speed of light c. If true, such a claim could have a deep impact for our understanding of physics.

The purpose of this short note is to study a possible source of error in the OPERA experiment, related to the satellite's motion, and suggest some straightforward tests and checks of the anomaly origin based upon the celebrated theory of General Relativity.

2 The eccentricity correction

General relativity introduces several subtle corrections to the GPS timing and clock syncronization beyond the usual time dilation effects[1, 2]. We will focus on the non-circular orbital correction in General Relativity.

2.1 Calculation

We follow [1] and review how the eccentricity correction arises.

$$\Delta_{ECC} = \int_{path} d\tau \left[-2\frac{GM_E}{c^2}\left(\frac{1}{a} - \frac{1}{r}\right)\right] \tag{1}$$

Integrating, we get, taking $ds/c \simeq dt$

$$\Delta_{ECC} = \int \left[2\frac{GM_E}{c^2}\left(\frac{1}{r} - \frac{1}{a}\right)\right] \frac{ds}{c} \simeq 2\frac{GM_E}{c^2} \int \left(\frac{1}{r} - \frac{1}{a}\right) dt \tag{2}$$

or

$$\Delta_{ECC} = 2\frac{GM_E}{ac^2} \int dt \left(\frac{e\cos E}{1 - e\cos E}\right) = \frac{2\sqrt{GM_E a}}{c^2} e\left(\sin E - \sin E_0\right) \tag{3}$$

$$\Delta_{ECC} = \frac{2\sqrt{GM_E a}}{c^2} e\sin E + \text{constant} \tag{4}$$

and where we have considered that for a keplerian orbit $r = a(1 - \cos E)$ the eccentric anomaly is defined as the solution of the trascendental equation

$$E - e\sin E = \sqrt{\frac{GM_E}{a^3}}\,(t - t_p) \tag{5}$$

being t_p the coordinate time of the perigee passage. In performing the integral, we have use that (5) can be differentiated to obtain the useful result

$$\frac{dE}{dt} = \frac{\sqrt{GM_E/a^3}}{1 - e\cos E} \tag{6}$$

2.2 Discussion

There is no need to suppose a GPS satellite moves in circular orbits. Although orbits are usually assumued to be circular ($e = 0$), due to perturbations these orbits are merely *almost* circular in reality, with eccentricity, say about[1] $0.01 < e < 0.03$. The effect of eccentrical orbits on the satellite's velocity v_S and the gravitational potential Φ_S is to introduce some periodic variation because of the changes in the orbital radius r_S. Then, the time of the signal arrival is related to the time on the receiving satellite's signal in a non-circular orbit through certain mathematical expression generally called the eccentricity correction. The error introduced by the eccentricity in the orbit is given by the following simple expression:

$$\Delta_{ECC} = \frac{2\vec{r}_S \cdot \vec{v}_S}{c^2} \tag{7}$$

[1] Of course, the reason to select the given range of the eccentricity is to match the observed delay. Thus, it is a fundamental test of our proposal to know if the given interval corresponds to the physical set-up in the OPERA experiment. We think it can be done with the GPS data.

with the GPS satellite position and velocity, $\vec{r}_S, \vec{v}_S$, measured in the ECI (Earth Centered Inertial frame) at time t. These vectors can be easily computed and calculated by plugging the orbital elements or parameters, and then, recast the previous equation into:

$$\Delta_{ECC} \simeq 2 \cdot \frac{\sqrt{GM_E a}}{c^2} e \sin E(t) \tag{8}$$

where G is the Newton's constant, M_E is the Earth mass, e is the eccentricity and $E(t)$ the eccentric anomaly at GPS time t and semimajor axis a (generally GPS are on a approximately geostationary orbit, i.e., $a \simeq R_{geostat} = 26562\text{km}$). Numerically, this error produces a periodic time delay about:

$$\Delta t_{delay} = 4.4428 \cdot 10^{-10} e \sin E \frac{\sqrt{a(m)}}{\sqrt{1\text{m}}} \text{s} \tag{9}$$

These quantities are computable by the broadcasted location in time and space of the satellite, and thus the GPS clocks beat the correct rate for observers on the Earth's surface. Hence, we remark that the precise time determination is possible *with GPS receivers* and must be taken into account by the proper software. Otherwise, they produce an error. Taking a reasonable interval for the eccentricity to be $0.01 \lesssim e \lesssim 0.03$, the time delay range is:

$$23\text{ns} \lesssim \Delta_{ECC} \lesssim 69\text{ns} \tag{10}$$

It is remarkable that the OPERA time delay between neutrinos and photons is in this range. There is an alternative way to understand the source of this error to show the consistence of our calculations. We compute the length the satellite moves in the time, measured in the ECI frame, in which the electromagnetic signal is emitted

$$L_S = v_S t = v_S \left(\frac{d}{c}\right) \tag{11}$$

Therefore the time correction due to the variation in the position of the GPS satellite would be:

$$\delta t = \frac{L_S}{c} = \frac{v_S d}{c^2} \tag{12}$$

where $d = 732\text{km}$ is the distance between CERN and Gran Sasso National Laboratory, v_S is the satellite's velocity and c is the speed of light. For a full round trip, the delay in time would be:

$$\delta T = 2\delta t = 2\frac{v_S d}{c^2} \tag{13}$$

Returning to our eccentricity correction, a non-circular orbit implies the position vector and the velocity of the satellite are not exactly orthogonal (in that case, the scalar product would vanish). Calling Θ to that angle, we get from (7)

$$\Delta_{ECC} = \frac{2 v_S r_S \cos\Theta}{c^2} \tag{14}$$

If we compare this equation with (13), we obtain that

$$d = 732\text{km} = r_S \times \cos\Theta. \tag{15}$$

With $r_S \approx 26562$km we get $\Theta \approx 88.42^\circ < 90^\circ$. This angle could be measured to provide a test of our proposal. Moreover, the correction of this error must be done by the user segment, and it is not clear for the present author if the analysis of OPERA data includes this correction made by the receiver to take the satellite's motion into account.

3 Summary and future tests

OPERA results are challenging and deserves a clear explanation. We have proposed a relatively minimal solution in terms of a General Relativity correction not-taken into account by the experimental analysis(as far as we know). This idea can be easily checked and falsified. We suggest:

- Search for a periodic variation of the observed (extracted) time delay by OPERA(and also similar experiments like MINOS) for the photon and neutrino time of flight. Compare the periodicity that might be obtained with the one induced by the orbital parameters in the satellite's orbit.
- Analyse in more detail the complete General Relativity corrections (such as Sagnac effect and rotation of Earth). Note that Shapiro delay is usually less important: it is a subnanosecond correction.
- Check if the total time delay caused by the leading general relativity corrections could indeed explain the data. A GPS system is a very accurate tool, but we must know if it implements all the effects that it can measure in our experimental devices (clocks and rulers).

Before we try a New Physics or exotic explanation of the OPERA results, like those in [5, 7, 8], we should be completely sure that not only special relativity but also general relativity are not able to explain the anomaly. Hints point to the contrary. It is quite difficult to adjust the observed difference of velocities to Lorentz violation, Quantum Gravity or some other ideas like tachyonic or faster than light motion in such a way that astrophysical observations [9] and other neutrino experiments be fully compatible. The conclusion seems to be that the likely explanation on the OPERA experiment is to consider the leading corrections to the time-of-flight induced by Einstein's General Relavity in the line of the present paper or the one in [3]. This situation is not new. As remarked by [4, 6], it also happened ten years ago in a similar way with light propagation in a dense medium.

In summary, we suggest OPERA to carefully reanalyse data taking into account the full leading corrections by General Relativity induced by both, the rotation of Earth *and* the satellite's orbit (non-circular). In the case General Relativity can not explain the observed delay and that new experiments(perhaps at MINOS and T2K) provide a similar delay, then and only then we should invoke and search for new physics. The more reasonable solution, in our opinion, would be then a refraction index effect on the light propagation, more than a superluminal likely "dark" neutrino. However, before we consider the "impossible" unlikely option, we must try every possible explanation. We are hoping to observe real New Physics phenomena, and OPERA experiment might be a possible (not confirmed) new physics signal provided *every current theory* shows to be unable to offer the answer.

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