Seesawlogy

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1 Introduction

One of the big issues of Standard Model (SM) is the origin of mass (OM). Usually, the electroweak sector implements mass in the gauge and matter sector through the well known Higgs mechanism. However, the Higgs mechanism is not free of its own problems. It is quite hard to assume that the same mechanism can provide the precise mass and couplings to every quark and lepton. Neutrinos, originally massless in the old-fashioned SM, have been proved to be massive. The phenomenon of neutrino mixing , a hint of beyond the SM physics(BSM), has been confirmed and established it, through the design and performing of different nice neutrino oscillation experiments in the last 20 years(firstly from solar neutrinos). The nature of the tiny neutrino masses in comparison with the remaining SM particles is obscure. Never a small piece of matter has been so puzzling, important and surprising, even mysterious. The little hierarchy problem in the SM is simply why neutrinos are lighter than the rest of subatomic particles. The SM can not answer that in a self-consistent way. If one applies the same Higgs mechanism to neutrinos than the one that is applied to quarks and massive gauge bosons, one obtains that their Yukawa couplings would be surprisingly small, many orders of magnitudes than the others. Thus, the SM with massive neutrinos is unnatural [1].

The common, somewhat minimal, solution is to postulate that the origin of neutrino mass is different and some new mechanism has to be added to complete the global view. This new mechanism is usually argued to come from new physics (NP). This paper is devoted to the review of the most popular (and somewhat natural) neutrino mass generation mechanism the seesaw, and the physics behind of it, the seesawlogy[2](SEE). It is organized as follows: in section 2, we review the main concepts and formulae of basic seesaws; next, in section 3 we study other kind of no so simple seesaws, usually with a more complex structure; in section 4, we discuss the some generalized seesaws called multiple seesaws; then, in section 5, we study how some kind of seesaw arises in theories with extra dimensions, and finally, we summarize and comment the some important key points relative to the the seesaws and their associated phenomenology in the conclusion, section 6.

[1] In the sense of 't Hooft's naturalness,i.e., at any energy scale μ, a set of parameters, $\alpha_i(\mu)$ describing a system can be small (natural), iff, in the limit $\alpha_i(\mu) \to 0$ for each of these parameters, the system exhibits an enhanced symmetry.

[2] Please, do not confuse the term with Sexology!

2 Basic seesawlogy

The elementary idea behind the seesaw technology (seesawlogy) is to generate Weinberg's dimension-5 operator $\mathcal{O}_5 = gL\Phi L\Phi$, where L represent a lepton doublet, using some tree-level heavy-state exchange particle that varies in the particular kind of the seesaw gadget implementation. Generally, then:

- Seesaw generates some Weinberg's dimension-5 operator $\mathcal{O}_5$, like the one above.
- The strength g is usually small. This is due to lepton number violation at certain high energy scale.
- The high energy scale, say Λ_s, can be lowered, though, assuming Dirac Yukawa couplings are small.
- The most general seesaw gadget *is* is through a set of n lefthanded (LH) neutrinos ν_L plus any number m of righthanded (RH) neutrinos ν_R written as Majorana particles in such a way that $\nu_R = \nu_L^c$.
- Using a basis (ν_L, ν_L^c) we obtain what we call the general $(n+m)\times(n+m)$ SEE matrix (SEX):

  $$M_\nu = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \tag{1}$$

  Here, M_L is a SU(2) triplet, M_D is a SU(2) doublet and M_R a SU(2) singlet. Every basic seesaw has a realization in terms of some kind of seesalogy matrix.

We have now several important particular cases to study, depending on the values that block matrices at (1) we select.

2.1 Type I

This realization correspond to the following matrix pieces:

- $M_L = 0$.
- M_D is a $(n \times m)$ Dirac mass matrix.
- M_N is a $(m \times m)$ Majorana mass matrix.
- Type I SEE lagrangian is given by (up to numerical prefactors)

  $$\mathcal{L}_S^I = \mathcal{Y}_{ij}^{Dirac}\bar{l}_{L_i}\tilde{\phi}\nu_{R_i} + M_{N_{ij}}\bar{\nu}_{R_i}\nu_{R_j}^c \tag{2}$$

  with $\phi = (\phi^+, \phi^0)^T$ being the SM scalar doublet, and $\tilde{\phi} = \sigma_2\phi\sigma_2$. Moreover, $\langle\phi^0\rangle = v_2$ is the vacuum expectation value (vev) and we write $M_D = \mathcal{Y}_D v_2$.

Now, the SEX M_ν is, generally, symmetric and complex. It can be diagonalized by a unitary transformation matrix $(n+m)\times(n+m)$ so $U^T M U = diag(m_i, M_j)$, providing us n light mass eigenstates (eigenvalues $m_i, i = 1, ..., n$) and m heavy eigenstates (eigenvalues $M_j, j = 1, ..., m$). The effective light $n \times n$ neutrino mass submatrix will be after diagonalization:

$$m_\nu = -M_D M_N^{-1} M_D^T \tag{3}$$

This is the basic matrix structure relationship for type I seesaw. Usually one gets commonly, if $M_D \sim 100$GeV,and $M_N = M_R \sim 10^{16} \sim M_{GUT}$, i.e., plugging these values in the previous formula we obtain a tipically small LH neutrino mass about $m_\nu \sim$ meV. The main lecture is that in order to get a small neutrino mass, we need either a very small Yukawa coupling or a very large isosinglet RH neutrino mass.

The general phenomenology of this seesaw can substancially vary. In order to get, for instance, a TeV RH neutrino, one is forced to tune the Yukawa coupling to an astonishing tiny value, typically $\mathcal{Y}_D \sim 10^{-5} - 10^{-6}$. The result is that neutrino CS would be unobservable (at least in LHC or similar colliders). However, some more elaborated type I models prevent this to happen including new particles, mainly through extra intermediate gauge bosons w', Z'. This type I modified models are usually common in left-right (LR) symmetric models or some Gran Unified Theories (GUT) with SO(10) or E_6 gauge symmetries, motivated due to the fact we *can not* identify the seesaw fundamental scale with Planck scale. Supposing the SM holds up to Planck scale with this kind of seesaw would mean a microelectronvolt neutrino mass, but we do know from neutrino oscillation experiments that the difference mass squared are well above the microelectronvolt scale. Therefore, with additional gauge bosons, RH neutrinos would be created by reactions $q\bar{q}' \rightarrow W'^{\pm} \rightarrow l^{\pm}N$ or $q\bar{q} \rightarrow Z'^0 \rightarrow NN(or \quad \nu N)$. Thus, searching for heavy neutrino decay modes is the usual technique that has to be accomplished in the collider. Note, that the phenomenology of the model depends on the concrete form gauge symmetry is implemented. In summary, we can say that in order to observe type I seesaw at collider we need the RH neutrino mass scale to be around the TeV scale or below and a strong enough Yukawa coupling. Some heavy neutrino signals would hint in a clean way, e.g., in double W' production and lepton number violating processes like $pp \rightarrow W'^{\pm}W'^{\pm} \rightarrow l^{\pm}l^{\pm}jj$ or the resonant channel $pp \rightarrow W'^{\pm} \rightarrow l^{\pm}N^* \rightarrow l^{\pm}l^{\pm}jj$.

2.2 Type II

The model building of this alternative seesaw is different. One invokes the following elements:

- A complex SU(2) triplet of (heavy) Higgs scalar bosons, usually represented as $\Delta = (H^{++}, H^{+}, H^{0})$.
- Effective lagrangian SEE type II

$$\mathcal{L}_S^{II} = \mathcal{Y}_{L_{ij}} l_i^T \Delta C^{-1} l_j \tag{4}$$

  where C stands for the charge conjugation operator and the SU(2) structure has been omitted. Indeed, the mass terms for this seesaw can be read from the full lagrangian terms with the flavor SU(2) structure present:

$$\mathcal{L}_S^{II} = -Y_\nu l_L^T C i\sigma_2 \Delta l_L + \mu_D H^T i\sigma_2 \Delta^{+} H + h.c. \tag{5}$$

Moreover, we have also the minimal type II seesawlogy matrix made of a scalar triplet:

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} \tag{6}$$

- $M_L = \mathcal{Y}_L v_3$, with $v_3 = \langle H^0 \rangle$ the vev rising the neutral Higgs a mass. Remarkably, one should remember that non-zero vev of SU(2) scalar triplet has an effect on the ρ parameter in the SM, so we get a bound $v_3 \lesssim 1\text{GeV}$.
- In this class of seesaw, the role of seesawlogy matrix is played by the Yukawa matrix $\mathcal{Y}_\nu$, a 3×3 complex and symmetric matrix, we also get the total leptonic number broken by two units($\Delta L = 2$) like the previous seesaw and we have an interesting coupling constant μ_D in the effective scalar potential. Minimization produces the vev value for Δ $v_3 = \mu_D v_2^2/\sqrt{2}M_\Delta^2$ and v_2 is give as before.

Then, diagonalization of Yukawa coupling produces:

$$M_\nu = \sqrt{2}\mathcal{Y}_\nu v_3 = \frac{\mathcal{Y}_\nu \mu_D v_2^2}{M_\Delta^2} \tag{7}$$

This seesawlogy matrix scenario is induced, then, by electroweak symmetry breaking and its small scale is associated with a large mass M_Δ. Again, a juidicious choice of Yukawa matrix elements can accomodate the present neutrino mass phenomenology. From the experimental viewpoint, the most promising signature of this kind of seesawlogy matrix is, therefore, the doubly charged Higgs. This is interesting, since this kind of models naturally give rise to $M_\Delta = M_{H^{++}}$, and with suitable mass, reactions like $H^{\pm\pm} \to l^\pm l^\pm$,$H^{\pm\pm} \to W^\pm W^\pm$,$H^\pm \to W^\pm Z$ or $H^+ \to l^+\bar{\nu}$.

2.3 Type III

This last basic seesaw tool is similar to the type I. Type II model building seesaw is given by the following recipe:

- We replace the RH neutrinos in type I seesaw by the neutral component of an $SU(2)_L$ fermionic triplet called σ, with zero hypercharge ($Y_\Sigma = 0$), given by the matrix

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix} \tag{8}$$

- Picking out m different fermion triplets, the minimal elements of seesaw type III are coded into an effective lagrangian:

$$\mathcal{L}_S^{III} = \mathcal{Y}_{ij}^{Dirac} \phi^T \bar{\Sigma}_i^c L_j - \frac{1}{2} M_{\Sigma_{ij}} \text{Tr}(\bar{\Sigma}_i \Sigma_j^c) + h.c. \tag{9}$$

- Effective seesawlogy matrix, size $(n+m)\times(n+m)$, for type III seesaw is given by:

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_\Sigma \end{pmatrix} \tag{10}$$

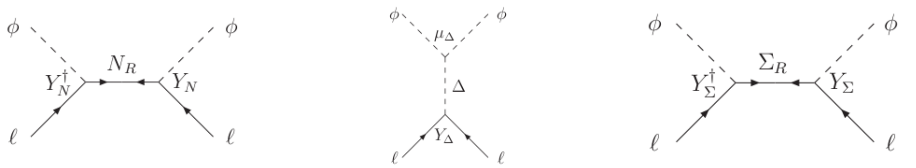

Figure 1: The 3 basic seesaw mechanisms. a) Type I. On the left. Heavy Majorana neutrino exchange. b) Type II. In the center. Heavy SU(2) scalar triplet exchange. c) Type III. Heavy SU(2) fermion triplet exchange.

Diagonalization of seesawlogy matrix gives

$$m_\nu = -M_D^T M_\Sigma^{-1} M_D \tag{11}$$

As before, we also get $M_D = \mathcal{Y}_D v_2$ and similar estimates for the small neutrino masses, changing the RH neutrino by the fermion triplet. Neutrino masses are explained, thus, by either a large isotriplet fermion mass M_Σ or a tiny Yukawa $\mathcal{Y}_D$. The phenomenology of this seesawlogy matrix scheme is based on the observation of the fermion triplet, generically referred as $E^\pm \equiv \Sigma^\pm, N \equiv \Sigma^0$, and their couplings to the SM fields. Some GUT arguments can make this observation plausible in the TeV scale (specially some coming from SU(5) or larger groups whose symmetry is broken into it). Interesting searches can use the reactions $q\bar{q} \to Z^*/\gamma^* \to E^+E^-$, $q\bar{q}' \to W^* \to E^\pm N$. The kinematical and branching ratios are very different from type II.

3 Combined seesaws

Different seesaw can be combined or the concept extended. This section explains how to get bigges SEE schemes.

3.1 Type I+II

The lagrangian for this seesaw reads:

$$-\mathcal{L}_m = \frac{1}{2}\overline{(\nu_L\ N_R^c)} \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + h.c. \tag{12}$$

where $M_D = \mathcal{Y}_\nu v/\sqrt{2}$, $M_L = \mathcal{Y}_\Delta v_\Delta$ and $< H >= v/\sqrt{2}$. Standard diagonalization procedure gives:

$$M_\nu = \begin{pmatrix} \hat{M}_\nu & 0 \\ 0 & \hat{M}_N \end{pmatrix} \tag{13}$$

If we consider a general 3+3 flavor example, $\hat{M}_\nu = diag(m_1, m_2, m_3)$ and also $\hat{M}_N = diag(M_1, M_2, M_3)$. In the so-called leading order approximation, the leading order seesaw mass formula for I+II seesawlogy matrix type is:

$$m_\nu = M_L - M_D M_R^{-1} M_D^T \tag{14}$$

Type I and type II seesaw matrix formulae can be obtained as limit cases of this combined case. Some further remarks:

- Both terms in the I+II formulae can be comparable in magnitude.
- If both terms are small, their values to the seesawlogy matrix may experiment significant interference effects and make them impossible to distinguish between a II type and I+II type.
- If both terms are large, interference can be destructive. It is unnatural since we obtain a small quantity from two big numbers. However, from phenomenology this is interesting since it could provide some observable signatures for the heavy Majorana neutrinos.

3.2 Double seesaw

A somewhat different seesaw structure in order to understand the small neutrino masses is got adding additional fermionic singlets to the SM. This is also interesting in the context of GUT or left-right models. Consider the simple case with one extra singlet(left-right or scalar under the gauge group, unlike the RH neutrino!). Then we obtain a 9×9 seesaw matrix structure as follows:

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_S \\ 0 & M_S^T & \mu \end{pmatrix} \tag{15}$$

The lagrangian, after adding 3 RH neutrinos, 3 singlets S_R and one Higgs singlet Φ follows:

$$\mathcal{L}_{double} = \bar{l}_L \mathcal{Y}_l H E_R + \bar{l}_L \mathcal{Y}_\nu \bar{H} N_R + \bar{N}_R^c \mathcal{Y}_S \Phi S_R + \frac{1}{2}\bar{S}_L^c M_\mu S_R + h.c. \tag{16}$$

The mass matrix term can be read from

$$-\mathcal{L}_m = \frac{1}{2}\overline{(\nu_L \; N_R^c \; S_R^c)} \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_S \\ 0 & M_S^T & \mu \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \\ S_R \end{pmatrix} \tag{17}$$

and where $M_D = \mathcal{Y}_\nu < H >$, and $M_S = \mathcal{Y}_S < \Phi >$. The zero/null entries can be justified in some models (like strings or GUTs) and, taking $M_S >> M_D$ the effective mass, after diagonalization, provides a light spectrum

$$m_\nu = M_D M_S^{T^{-1}} \mu M_S^{-1} M_D^T \tag{18}$$

When $\mu >> M_S$ the extra singlet decouples and show a mass structure $m_S = M_S \mu^{-1} M_S^T$, and it can be seen as an effective RH neutrino mass ruling a type I seesaw in the $\nu_L - \nu_L^c$ sector. Then, this singlet can be used as a "phenomenological bridge" between the GUT scale and the $B-L$ usual scale (3 orders below the GUT scale in general). This double structure of the spectrum in the sense it is doubly suppressed by singlet masses and its double interesting limits justifies the name "double" seesaw. The *inverse type I* is a usual name for the double seesaw too in some special parameter values. Setting $\mu = 0$, the global lepton number $U(1)_L$ is conserved and the neutrino are massless. Neutrino masses go to zero values, reflecting the restoration of global lepton number conservation. The heavy sector would be 3 pairs of pseudo-Dirac neutrinos, with CP-conjugated Majorana components and tiny mass splittings aroung μ scale. This particular model is very interesting since it satisfies the naturalness in the sense of 't Hooft.

3.3 Inverse type III

It is a inverse plus type III seesawlogy matrix combination. We use a (ν_L, Σ, S) basis, and we find the matrix

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_\Sigma & M_S \\ 0 & M_S^T & \mu \end{pmatrix} \tag{19}$$

Like the previous inverse seesaw, in the limit $\mu \to 0$, the neutrino mass is small and suppressed. The Dirac Yukuwa coupling strength may be adjusted to order one, in contrast to the normal type III seesawlogy matrix. This mechanism has some curious additional properties:

- The charged lepton mass read off from the lagrangian is:

$$M_{lep} = \begin{pmatrix} M_l & M_D \\ 0 & M_\Sigma \end{pmatrix} \tag{20}$$

- After diagonalization of M_{lep}, size $(n+m) \times (n+m)$, the $n \times n$ coupling matrix provide a neutral current (NC) lagrangian, and since the matrix shows to be nonunitary, this violates the Glashow-Iliopoulos-Maiani (GIM) mechanism and sizeable tree level flavor-changing neutral currents appear in the charged lepton sector.

3.4 Linear seesaw

Other well known low-scale SEE variant is the so-called linear seesaw. It uses to arise from $SO(10)$ GUT and similar models. In the common (ν, ν^c, S) basis, the seesawlogy matrix can be written as follows:

$$M_\nu = \begin{pmatrix} 0 & M_D & M_L \\ M_D^T & 0 & M_S \\ M_L^T & M_S^T & 0 \end{pmatrix} \tag{21}$$

The lepton number conservation is broken by the term $M_L \nu S$, and the effective light neutrino mass, after diagonalization, can be read from the next expression

$$M_\nu = M_D (M_L M^{-1})^T + (M_L M^{-1}) M_D^T \tag{22}$$

This model also suffers the same effect than the one in the inverse seesaw. That is, in the limit $M_L \to 0$, neutrino mass goes to zero and the theory exhibit naturalness. The name linear is due to the fact that the mass dependence on M_D is linear and not quadratic, like other seesaw.

4 Multiple seesaws

In (libro 2011) and references therein, a big class of multiple seesaw models were introduced. Here we review the basic concepts and facts, before introduce the general formulae for multiple seesaws(MUSE):

- Main motivation: MUSEs try to satisfy both naturalness and testability at TeV scale, in contrast with other basic seesaw. Usually, a terrible fine-tuning is required to implement seesaw, so that the ratio M_D/M_R and the Yukawa couplings can be all suitable for experimental observation, such as new particles or symmetries. This fine-tuning between M_D and M_R is aimed to be solved with MUSEs.

- Assuming a naive electroweak seesaw so that $m \sim (\lambda\Lambda_{EW})^{n+1}/\Lambda_S^n$, where λ is a Yukawa coupling and n is an arbitrary integer larger than the unit, without any fine-tuning, one easily guesses:

$$\Lambda_S \sim \lambda^{\frac{n+1}{n}} \left[\frac{\Lambda_{EW}}{100\text{GeV}}\right]^{\frac{n+1}{n}} \left[\frac{0.1\text{eV}}{m_\nu}\right]^{1/n} 10^{\frac{2(n+6)}{n}}\text{GeV} \tag{23}$$

  Thus, MUSEs provide a broad clase of parameter ranges in which a TeV scale seesaw could be natural and testable.

- The most simple MUSE model at TeV scale is to to introduce some singlet of fermions S^i_{nR} and scalars Φ_n, with $i = 1, 2, 3$ and $n = 1, 2, \cdots$. This field content can be realized with the implementation of global $U(1) \times Z_{2N}$ gauge symmetry leads to two large classes of MUSEs with nearest-neighbours interaction matrix pattern. The first class owns an even number of S^i_{nR} and Φ_n and corresponds to a straightforward extension of the basic seesaw. The second class has an odd number of S^i_{nR} and Φ_n, and it is indeed a natural extension of the inverse seesaw.

- The phenomenological lagrangian giving rise to MUSEs is:

$$-\mathcal{L}_\nu = \bar{l}_L \mathcal{Y}_\nu \tilde{H} N_R + \bar{N}^c_R \mathcal{Y}_{S_1} S_{1R}\Phi_1 + \sum_{i=2}^{n} \overline{S^c_{(i-1)R}} \mathcal{Y}_{S_i} S_{iR}\Phi_i + \\ + \frac{1}{2}\overline{S^c_{nR}} M_\mu S_{nR} + h.c. \tag{24}$$

  Here $\mathcal{Y}_\nu$ and $\mathcal{Y}_{S_i}$ are the 3x3 Yukawa coupling matrices, and M_μ is a symmetric Majorana mass matrix. After spontaneous symmetry breaking(SSB), we get a $3(n+2) \times 3(n+2)$ neutrino mass matrix $\mathcal{M}$ in the flavor bases $(\nu_L, N^c_R, S^c_{1R}, ...S^c_{nR})$ and their respective charge-conjugated states, being

$$\mathcal{M} = \begin{pmatrix} 0 & M_D & 0 & 0 & 0 & \cdots & 0 \\ M_D^T & 0 & M_{S_1} & 0 & 0 & \cdots & 0 \\ 0 & M_{S_1}^T & 0 & M_{S_2} & 0 & \cdots & 0 \\ 0 & 0 & M_{S_2}^T & 0 & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & M_{S_{n-1}} & 0 \\ \vdots & \vdots & \vdots & \cdots & M_{S_{n-1}}^T & 0 & M_{S_n} \\ 0 & 0 & 0 & \cdots & 0 & M_{S_n}^T & M_\mu \end{pmatrix} \tag{25}$$

  where we have defined $M_D = \mathcal{Y}_\nu < H >$ and $M_{S_i} = \mathcal{Y}_{S_i} < \Phi_i >$, $\forall i, i = 1, ..., n$, and they are 3×3 matrices each of them. Note that

Yukawa terms exist only if $|i-j|=1, \forall i,j=0,1,...,n$ and that $\mathcal{M}$ can be written in block-form before diagonalization as

$$\mathcal{M} = \begin{pmatrix} 0 & \tilde{M}_D \\ \tilde{M}_D^T & \tilde{M}_\mu \end{pmatrix} \tag{26}$$

with $\tilde{M}_D = (M_D \ 0)$ a $3\times 3(n+1)$ and $\tilde{M}_\mu$ a symmetric $3(n+1)\times 3(n+1)$ mass matrix.

- **General phenomenological features**: *non-unitary neutrino mixing* (in the submatrix boxes) and CP violation (novel effects due to non-unitarity or enhanced CP-phases), *collider signatures of heavy Majorana neutrinos* (class A MUSEs preferred channel $pp \to l_\alpha^\pm l_\beta^\pm X$, i.e., the dilepton mode; class B MUSEs, with $M_\mu << M_{EW}$, favourite channel is $pp \to l_\alpha^\pm l_\beta^\pm l_\gamma^\pm X$, i.e., the trilepton mode and the mass spectrum of heavy Majorana would consist on pairing phenomenon, showing nearly degenerate masses than can be combined in the so-called pseudo-Dirac particles).
- **Dark matter particles**. One or more of the heavy Majorana neutrinos and gauge-singlet scalars in our MUSE could last almost forever, that is, it could have a very long timelife and become a good DM candidate. It could be fitted to some kind of weakly interacting massive particle (WIMP) to build the cold DM we observe.

4.1 Class A

This MUSE is a genaralization of canonical SEE. MUSE A composition:

- Even number of gauge singlet fermion fields S_{nR}^i, $n=2k, \quad k=1,2,...,$.
- Even number of scalar fields Φ_n, $n=2k, \quad k=1,2,...,$.
- Effective mass matrix of the 3 light Majorana neutrinos in the leading approximation:

$$M_\nu = -M_D \left[\prod_{i=1}^{k}\left(M_{S_{2i-1}}^T\right)^{-1} M_{S_{2i}}\right] M_\mu^{-1} \left[\prod_{i=1}^{k}\left(M_{S_{2i-1}}^T\right)^{-1} M_{S_{2i}}\right]^T M_D^T \tag{27}$$

When $k=0$, we obviously recover the traditional SEE $M_\nu = -M_D^T M_R^{-1} M_D$ if we set $S_{0R}=N_R$ and $M_\mu = M_R$. Note that since the plugging of $M_{S_{2i}} \sim M_D \sim \mathcal{O}(\Lambda_{EW})$ and $M_{S_{2i-1}} \sim M_\mu \sim \mathcal{O}(\Lambda_{SEE})$, then $M_\nu \sim \Lambda_{EW}^{2(k+1)}/\Lambda_{SEE}^{2k+1}$, and hence we can effectively lower the usual SEE scale to the TeV without lacking testability.

4.2 Class B

This MUSE is a genaralization of inverse seesaw. MUSE B composition:

- Odd number of gauge singlet fermion fields S_{nR}^i, $n=2k+1, \quad k=1,2,...,$.
- Odd number of scalar fields Φ_n, $n=2k+1, \quad k=1,2,...,$.

- Effective mass matrix of the 3 light Majorana neutrinos in the leading approximation:

$$M_\nu = M_D \left[\prod_{i=1}^{k} \left(M^T_{S_{2i-1}}\right)^{-1} M_{S_{2i}}\right] \left(M^T_{S_{2k+1}}\right)^{-1}$$
$$\times M_\mu \left(M^T_{S_{2k+1}}\right)^{-1} \left[\prod_{i=1}^{k} \left(M^T_{S_{2i-1}}\right)^{-1} M_{S_{2i}}\right]^T M_D^T \tag{28}$$

When $k = 0$, we evidently recover the traditional inverse SEE but with a low mass scale M_μ: $M_\nu = M_D^T (M_{S_1}^T)^{-1} M_\mu (M_{S_1}^T)^{-1} M_D^T$. Remarkably, if $M_{S_{2i}} \sim M_D \sim \mathcal{O}(\Lambda_{EW})$ and $M_{S_{2i-1}} \sim \mathcal{O}(\Lambda_{SEE})$ hold $\forall i, i = 1, 2, ...k$, the mass scale M_μ is not necessary to be at all as small as the inverse SEE. Taking, for instance, $n = 3$, the double suppressed M_ν provides the ratios $M_D/M_{S_1} \sim \Lambda_{EW}/\Lambda_{SEE}$ and $M_{S_2}/M_{S_3} \sim \Lambda_{EW}/\Lambda_{SEE}$, i.e., $M_\nu \sim 0.1$eV results from $Y_\nu \sim Y_{S_1} \sim Y_{S_2} \sim Y_{S_3} \sim \mathcal{O}(1)$ and $M_\mu \sim 1$keV at $\Lambda_{SEE} \sim 1$TeV.

5 Extra dimensional relatives

Several authors have introduced and studied a higher-dimensional cousin of the seesaw and seesaw matrix. We consider a brane world theory with a 5d-bulk (volume), where the SM particles are confined to the brane. We also introduce 3 SM singlet fermions Ψ_i with $i = 1, 2, 3$. Being singlets, they are not restricted to the brane and can scape into the extra spacetime dimensions(EDs). The action responsible for the neutrino masses is given by

$$S = S_{bulk,5d} + S_{brane,4d} \tag{29}$$

with

$$S_{bulk,5d} = \int d^4x dy \left[i\overline{\Psi} \not{D} \Psi - \frac{1}{2}\left(\overline{\Psi^c} M_R \Psi + h.c.\right)\right] \tag{30}$$

and

$$S_{brane,4d} = \int_{y=0} d^4x \left[-\frac{1}{\sqrt{M_S}} \overline{\nu_L} m^c \Psi - \frac{1}{\sqrt{M_S}} \overline{\nu_L^c} m^c \Psi + h.c.\right] \tag{31}$$

After a KK procedure on a circle with radius R, we get the mass matrix for the n-th KK level

$$\mathcal{M}_n = \begin{pmatrix} M_R & n/R \\ n/R & M_R \end{pmatrix} \tag{32}$$

and a Dirac mass term with $m_D = m/\sqrt{(2\pi M_S R)}$. The KK tower is truncated at the level N, and we write the mass matrix in the suitable KK basis, to obtain:

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & m_D & m_D & m_D & \cdots & m_D \\ m_D^T & M_R & 0 & 0 & 0 & \cdots & 0 \\ m_D^T & 0 & M_R - \dfrac{1}{R} & 0 & 0 & \cdots & 0 \\ m_D^T & 0 & 0 & M_R + \dfrac{1}{R} & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ m_D^T & 0 & 0 & 0 & 0 & M_R - \dfrac{N}{R} & 0 \\ m_D^T & 0 & 0 & \cdots & 0 & 0 & M_R + \dfrac{N}{R} \end{pmatrix} \tag{33}$$

Note that M_R is not assumed to be in the electroweak scale and its value is free. We diagonalize the above matrix to get the light neutrino mass matrix:

$$m_\nu \simeq m_D \left(\sum_{n=-N}^{N} \frac{1}{M_R + n/R} \right) m_D^T = m_D \left(M_R^{-1} + \sum_{n=1}^{N} \frac{2M_R}{M_R^2 - n^2/R^2} \right) m_D^T \tag{34}$$

Already considered by some other references, the limit $N \to \infty$ produces the spectrum

$$m_\nu \simeq m_D \frac{\pi R}{\tan(\pi R M_R)} m_D^T \tag{35}$$

At level of the highest KK state, say N, the light neutrino mass becomes, neglecting the influence of lower states,

$$m_\nu \simeq m_D \left(\sum_{n=-N}^{N} \frac{1}{M_R + N/R} \right) m_D^T \tag{36}$$

Then, irrespectively the value of M_R, if $M_R << N/R$, the spectrum get masses that are suppressed by N/R, i.e., $m_\nu \simeq m_D m_D^T R/N$. Some further variants of this model can be built in a similar fashion to get different mass dependences on m_D (here quadratic).

6 Conclusion and outlook

The seesaw has a very interesting an remarkable structure and its a remarkable neutrino mass mechanism BSM. It gives a way to obtain small masses from a high energy cut-off scale, yet to find or adjust. Neutrino oscillation experiment hints that the seesaw fundamental scale is just a bit below of GUT scale, although, as this review has shown and remembered, the nature and value of that seesaw energy scale is highly model dependent: the seesawlogy matrix is a mirror of the GUT/higher gauge-symmetry involved in the small neutrino masses, the EW SSB and the particle content of the theory. Moreover, in spite of seesaw is the more natural way to induce light masses on neutrino(or even every particle using some *universal* seesaw), their realization in Nature is to be proved yet. In order to test the way, if any, in which seesaw is present experimental hints on colliders in the line of this article, DM searches and other neutrino experiments(like those in neutrino telescopes, neutrino superbeams or neutrino factories) will be pursued in present and future time. We live indeed in an exciting experimental era and the discovery of sterile neutrino is going to be, according to Mohapatra, a boost and most impactant event than the one a hypothetical Higgs particle finding will provoke. Their time is just running now.