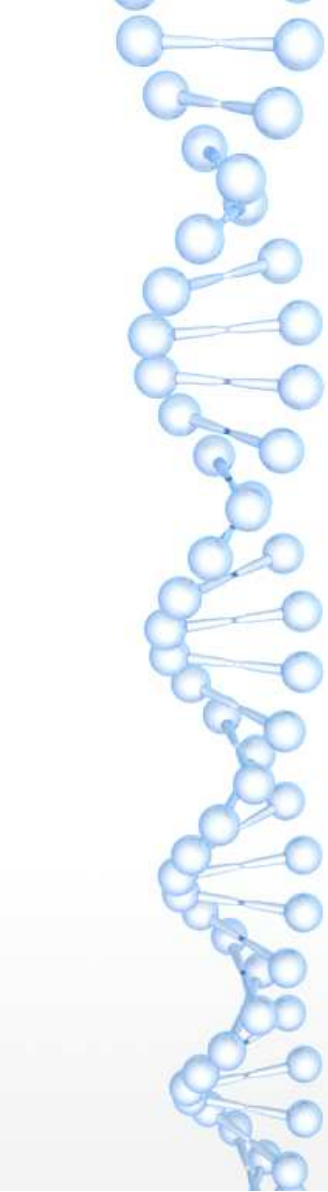
El Multiverso y las otras dimensiones: un viaje matemático y filosófico

	Electron ($R \simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Moon	Earth	Solar System ($1\mathrm{AU}/M_\odot G$)	Milky Way (visible)	Sun	Galaxy Cluster	Universe ($M \propto R^3$)
$R/(MG)$	0^+	7.1×10^{38}	2.0×10^{43}	2.4×10^{26}	3.2×10^{10}	1.4×10^{9}	1.0×10^{8}	1.5×10^{6}	4.7×10^{5}	$\sim 10^5$	0^+



Plan:

- Part (I): Understanding the (mathematical) multiverses.
- Part (II): Advanced stuff

PART(I)→ Introduction to the Multiverses: the mathematical reality hypothesis!

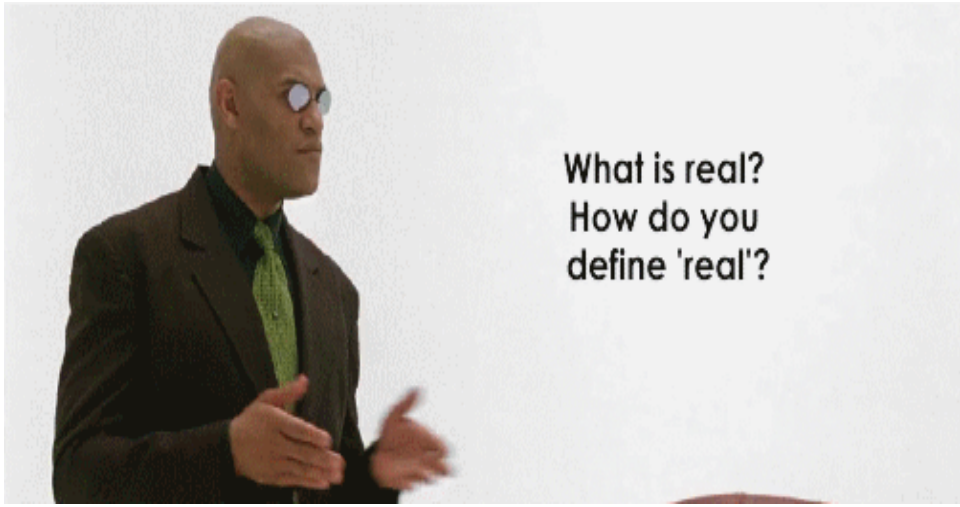
External Reality Hypothesis (ERH):

There exists an external physical reality completely independent of us humans.

↓

Mathematical Universe Hypothesis (MUH):

Or external physical reality is a mathematical structure.

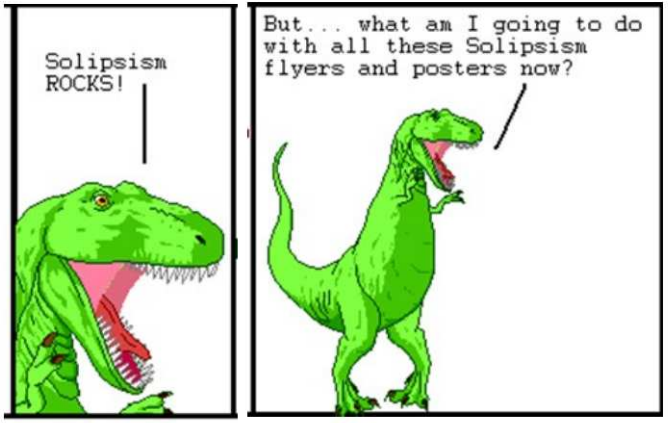


- Physmatics = Physics + Mathematics
- Physchematics = Physics + Chemistry + Mathemics

Solipsistists here?

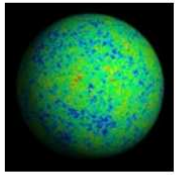
External Reality Hypothesis (ERH):

There exists an external physical reality completely independent of us humans.

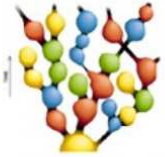


Where are the parallel universes?

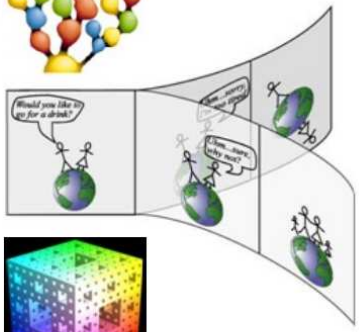
1) Far away in space



2) Infinitely far away in space



3) Elsewhere in Hilbert space

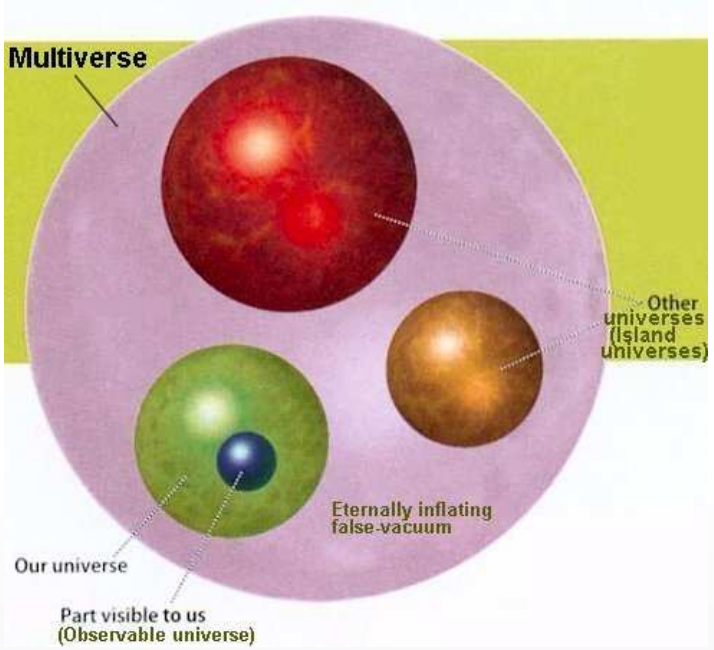


4) Elsewhere in "math space"

Niveles de Multiverso (Tegmark)/ Levels of the Multiverse

- Level I: Other Hubble volumes have different initial conditions
- Level II: Other post-inflation bubbles may have different effective laws of physics (constants, dimensionality, particle content)
- Level III: Other branches of the quantum wavefunction add nothing qualitatively new
- Level IV: Other mathematical structures have different fundamental equations of physics

- Pictures? A naive (not the only) one:



Multiverse: Levels 1 to 4

Level 1: Regions beyond our cosmic horizon

Features: Same laws of physics, different initial conditions
Assumptions: Infinite space, ergodic matter distribution
Evidence:
- Microwave background meaurements point to flat, infinite space, large-scale smoothness
- Simplest model

Level 2: Other post-inflation bubbles

Features: Same fundamental equations of physics, but perhaps different constants, particles and dimensionality
Assumption: Chaotic inflation occurred
Evidence:
- Inflation theory explains flat space, scale-invariant fluctuations, solves horiz on problem and monopole problems and can naturally explain such bubbles
- Explains fine-tuned parameters

Level 3: The Many Worlds of Quantum Physics

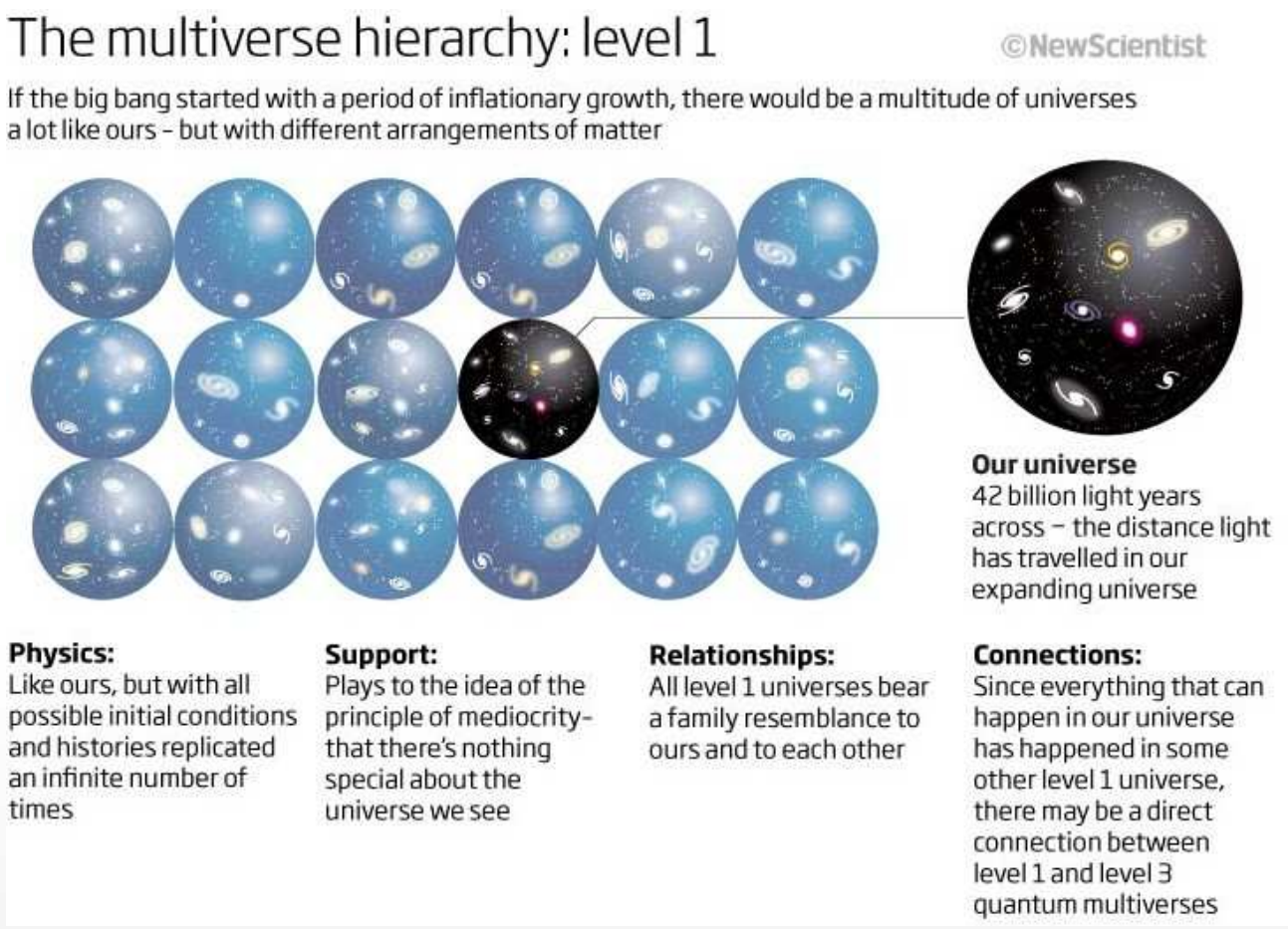
Features: Same as level 2
Assumption: Physics unitary
Evidence:
- Experimental support for unitary physics
- AdS/CFT correspondence suggests that even quantum gravity is unitary
- Decoherence experimentally verified
- Mathematically simplest model

Level 4: Other mathematical structures

Features: Different fundamental equations of physics
Assumption: Mathematical existence = physical existence
Evidence:
- Unreasonable effectiveness of math in physics
- Answers Wheeler/Hawking question: "why these equations, not others"

Multiverse: Level 1, details

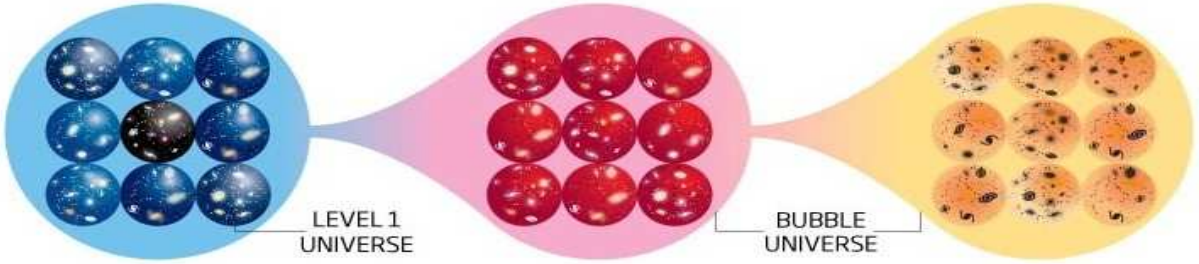
(Todo lo que puede ocurrir, ha ocurrido u ocurrirá)



Multiverse: Level 2 details
Key idea: inflation! (Money?) Well, no...

The multiverse hierarchy: level 2

In the theory of eternal inflation, the space between universes continues to expand, and a limitless number of new "bubble" universes, with very different properties, continue to form. Each bubble universe contains an infinite number of its own level 1 universes



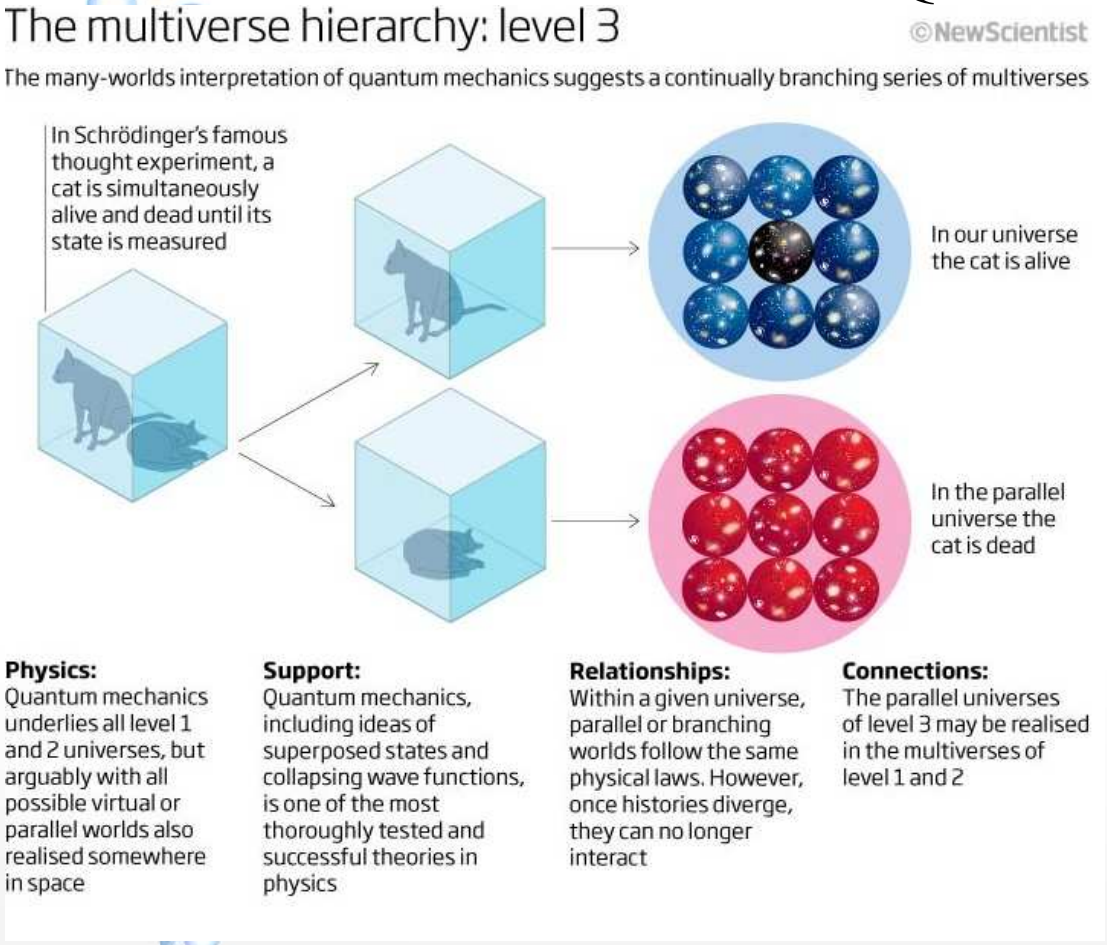
Physics:
Other bubble universes exhibit different laws of physics and have different dimensionality, particles, constants and forces to those seen in our universe. We might eventually discover that all these parameters flow from the same "theory of everything"

Support:
Inflation explains the uniformity and flatness of our universe and details of the cosmic microwave background. Eternal inflation implies bubble universes and provides a way of supplying string theory with the many universes it demands

Relationships:
Level 2 universes vary greatly. They represent separate bubbles or domains with different properties, and are separated from each other by inflating space

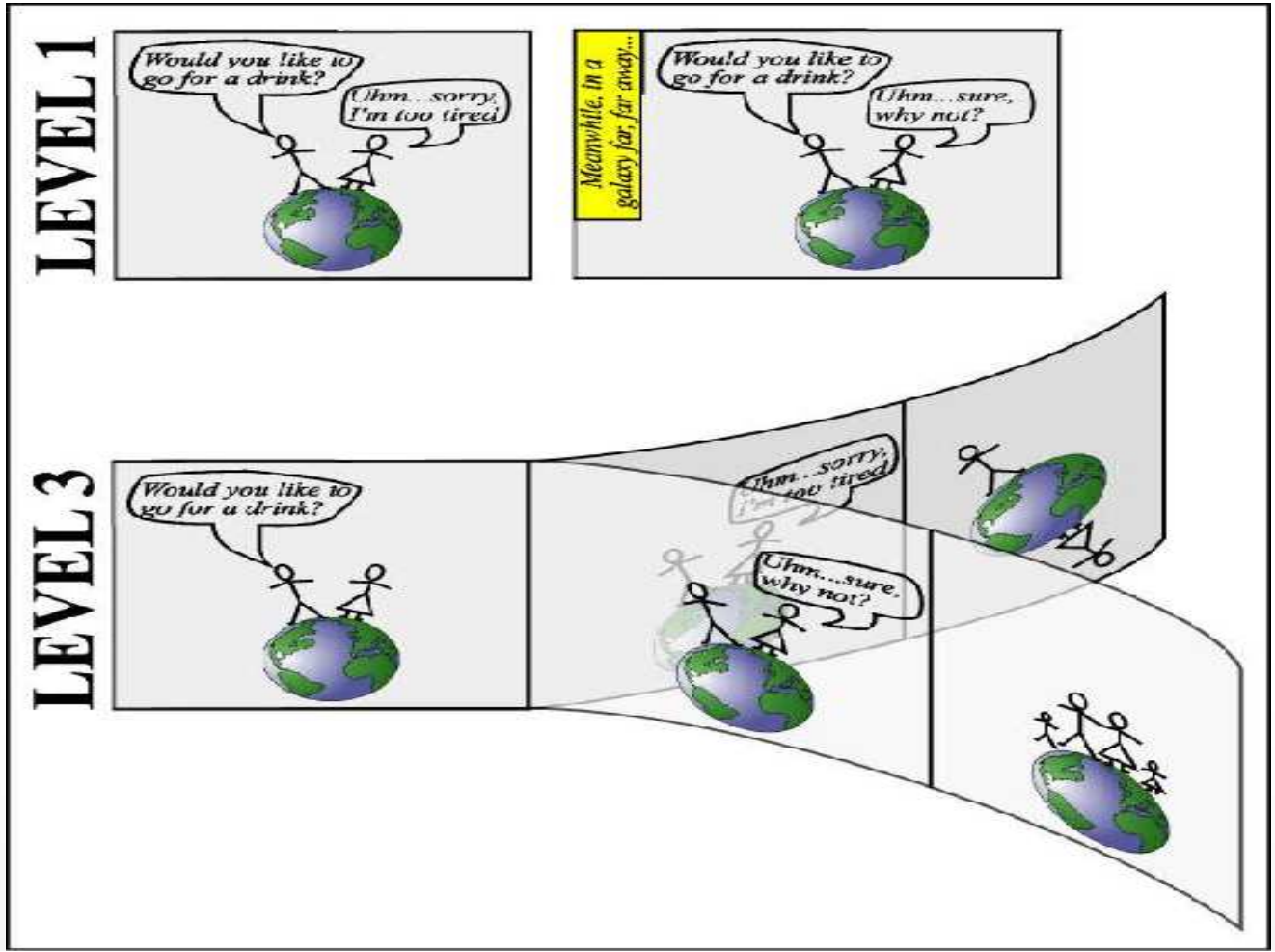
Connections:
Level 2 includes all possible level 1 universes plus an enormous variety of much stranger universes. Since everything that can happen in a particular level 2 universe has happened in other universes, level 2 may also correspond to the universes in the level 3 quantum multiverse

Multiverse: Level 3, details (I)
The Quantum realm...



- The wave function rises (Is it "real"? Not really...Generally, it is...complex!)
- $$\Psi \quad |\Psi|^2$$
- Big deal. The only real stuff are...Probabilities (real stuff, literally).

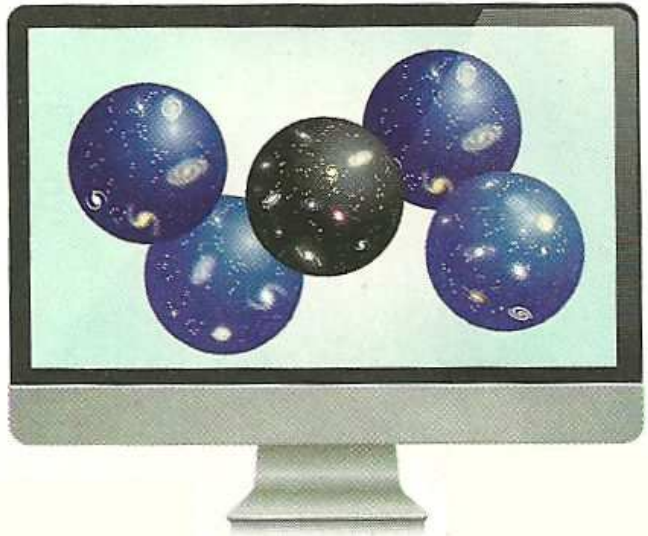
Multiverse 3 (and 1): in a joke...



Multiverse 4: Or when things go crazy...

The multiverse hierarchy: level 4

If our universe is just a simulation, there could be infinitely many kinds of universes that differ in arbitrary ways from ours



The multiverse hierarchy: level 4

The multiverse hierarchy: level 4

If our universe is just a simulation, there could be infinitely many kinds of universes that differ in arbitrary ways from ours

Physics:
Anything goes

Support:
None. The idea is that long-lived technological civilisations will probably command vast computing power and may choose to run multiple "ancestor simulations" which will soon outnumber natural universes. There may be a one-to-one relationship between mathematics and reality: every conceivable mathematical system may represent a real universe

Relationships:
Arbitrary or non-existent

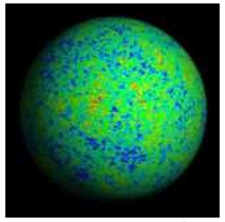
Connections:
Level 4 must contain the ultimate theory of everything. If so, there's no level 5

Multiversal review (I)

What are the 4 multiverse levels like?

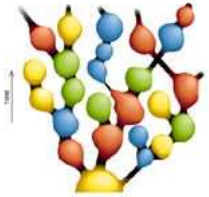
10^{100} bits?

1) Same effective laws of physics, different initial conditions



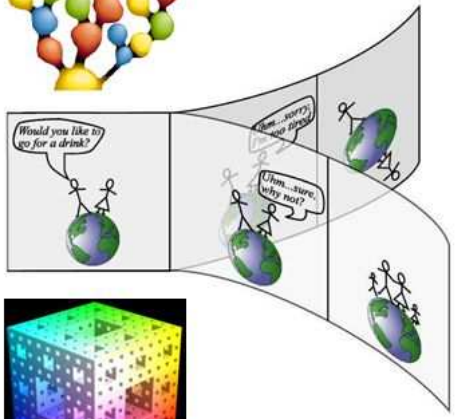
10^{3} bits?

2) Same fundamental laws of physics, different effective laws ("bylaws")



10^{2} bits?

3) Nothing qualitatively new



4) Different fundamental laws of physics

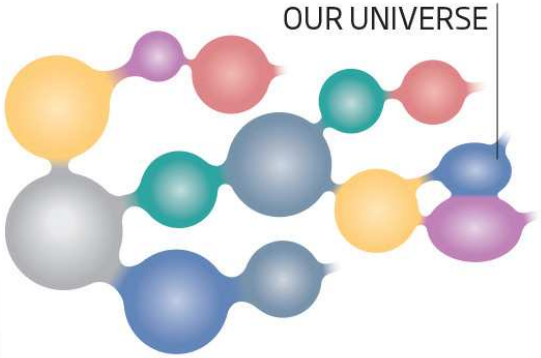
0 bits!

The bigger the multiverse, the simpler the theory.

Multiversal review (II)

Eternal inflation

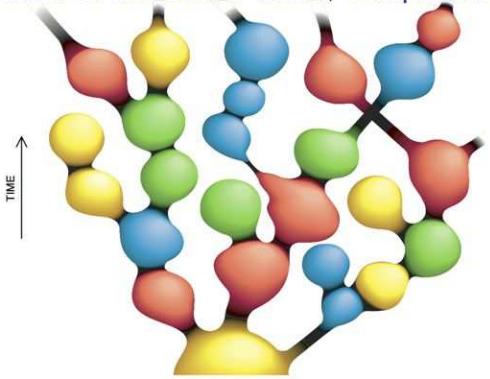
Our universe could be one of many bubble universes in a branching, ever-expanding multiverse. If we bump up against another universe, it could leave a mark on the ancient sky



Inflationary Multiverse

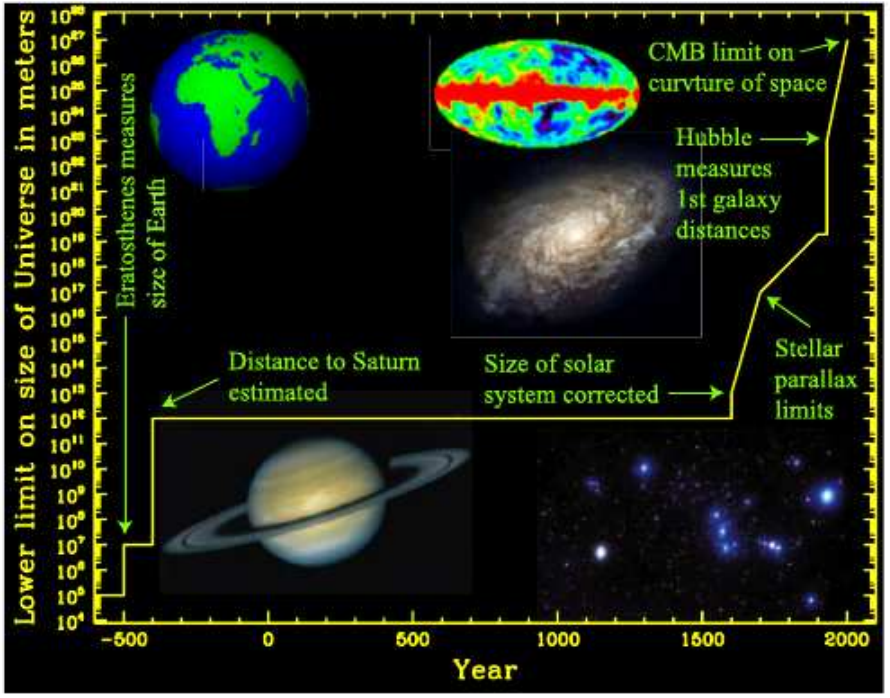
For a long time, people believed in the cosmological principle, which asserted that the universe is everywhere the same.

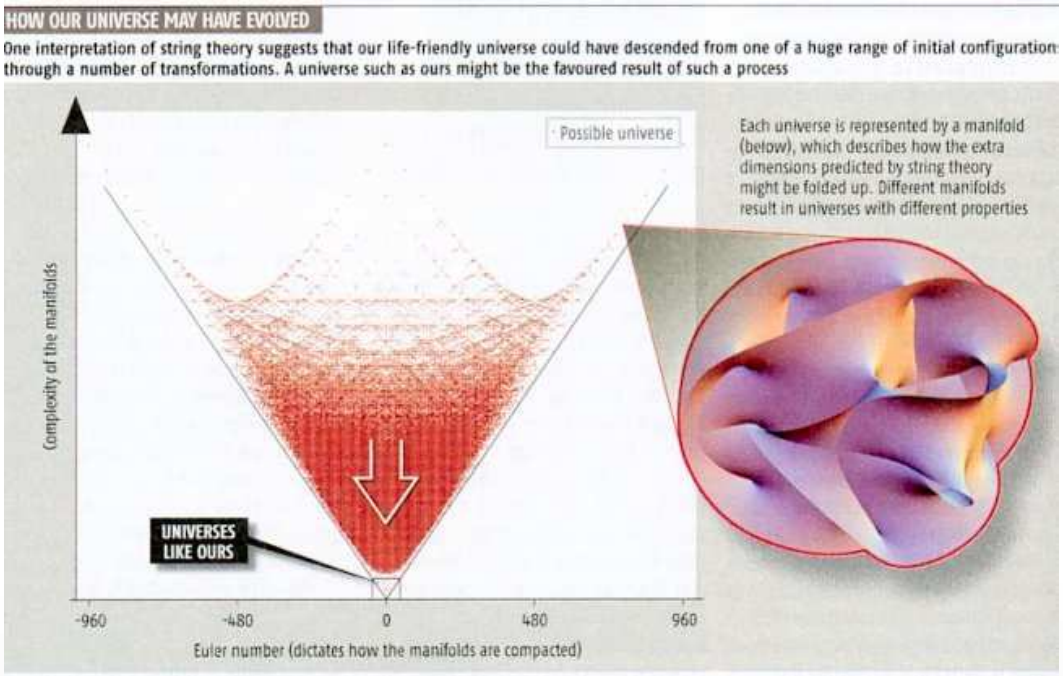
This principle is no longer required. Inflationary universe may consist of many parts with different properties depending on the local values of the scalar fields, compactifications, etc.



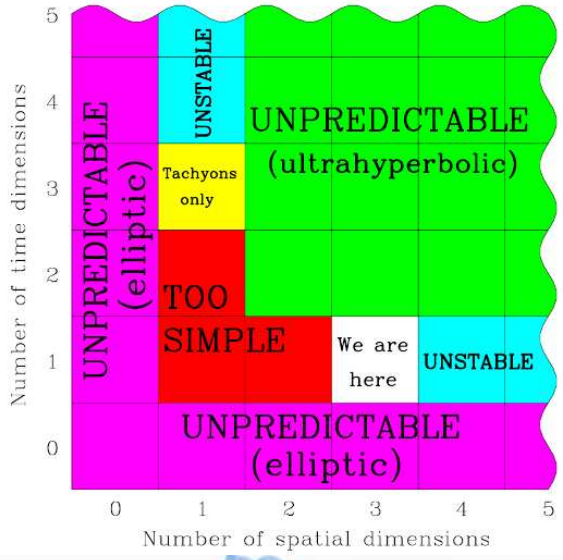
Multiversal review (III)

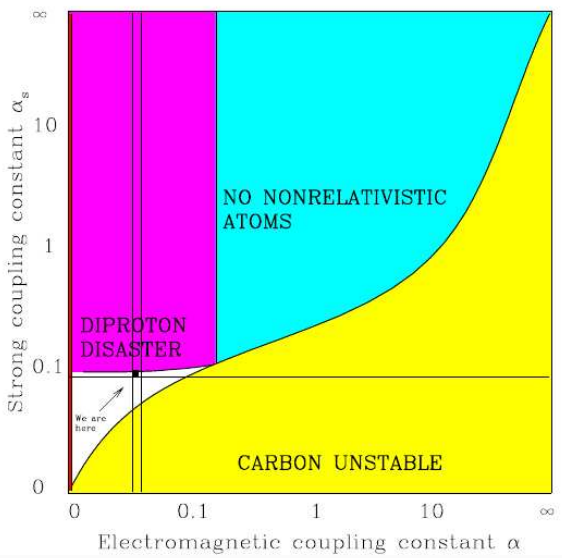
Multiverse: beyond. How many? More dimensions?

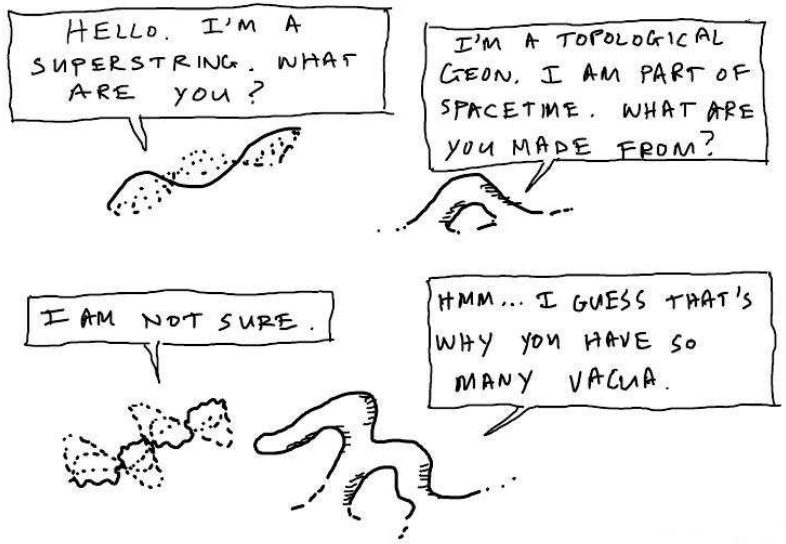




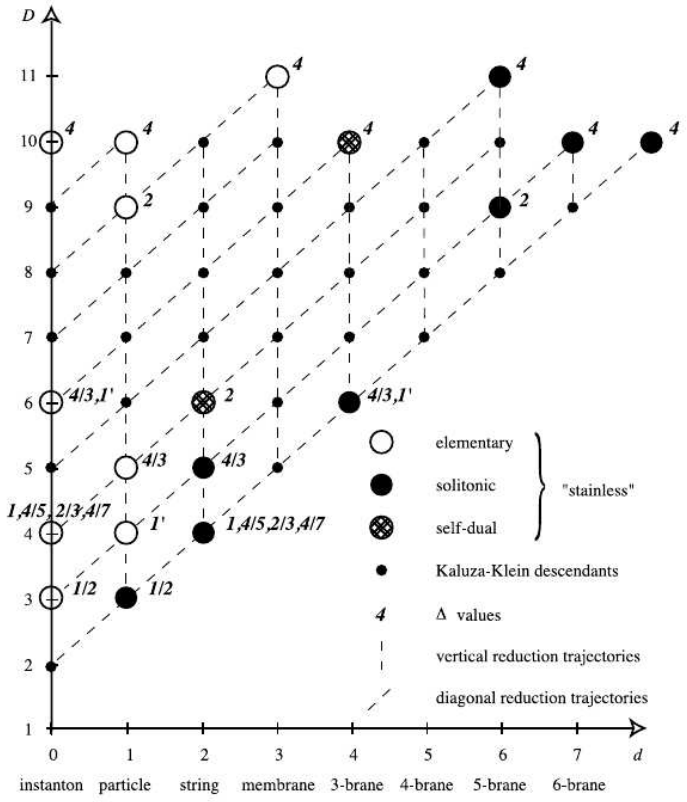
Part (II)→ The mathematical multidimensional multiverses (physmatical, physchematical!)







The views of superstrings and M-theory Dp-branes vs. The current Big Bang



"Brane-scan" of supergravity p-brane solutions, linked by worldvolume (diagonal) and transverse-space (vertical) dimensional reductions

Was the Big Bang the beginning of the universe?

	$\dim \mathcal{H} = \text{finite}$	$\dim \mathcal{H} = \infty$
time is emergent	universe had a beginning	universe may or may not have had a beginning
time is fundamental	universe is eternal, with a finite recurrence time (& Boltzmann brains)	universe is eternal, and need never experience recurrence

Dimensions and hyperspaces

- Geometry(Geometría):
- Vacuum, point, line, plane, space,...Hyperspace! (0,1,2,3,...D>3!)
- D=-1????? D=-n??? D=a+bi?????
- Generally: $D \geq 0$

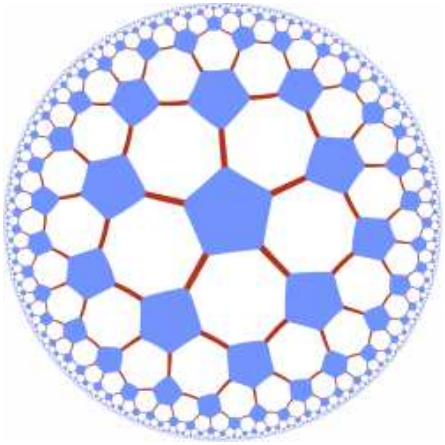
- Exceptionalities:
- a) Non-integer constant dimensions: fractals!
- b) Non-integer VARIABLE dimensions: multifractals!
- c) Negative dimensions
- (weird?)
- d) Complex dimensions?
- e) Something else????

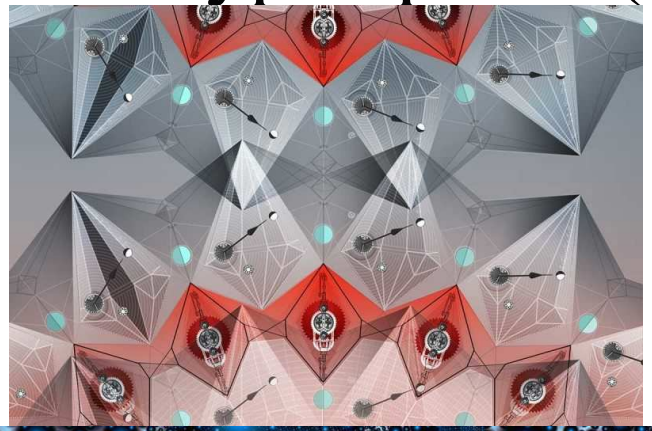
Dimensions and hyperspaces (II)

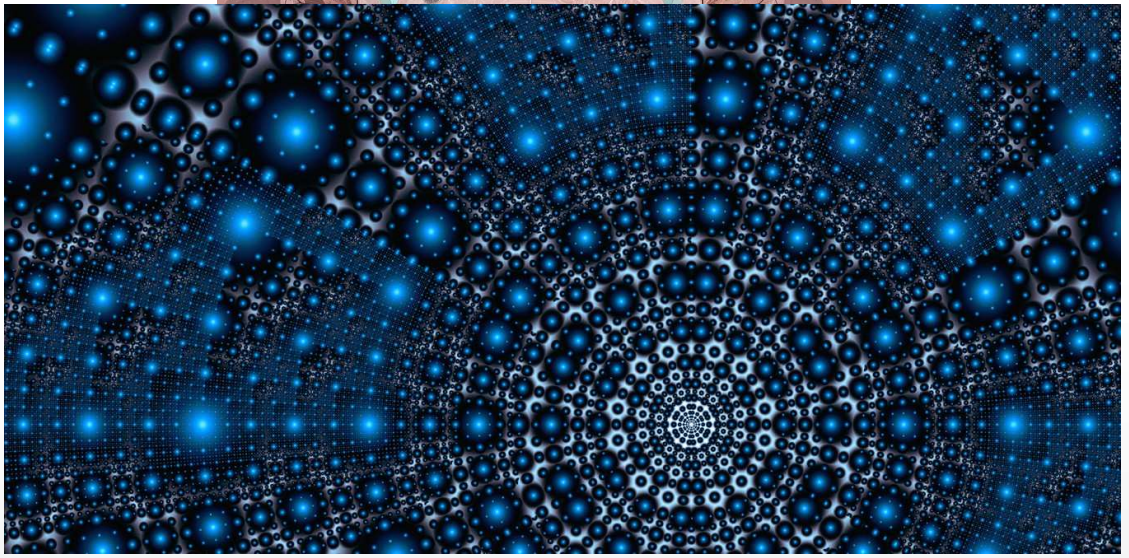
- Void = Vacuum = Nothingless (boring?)
- Points: discrete (lattice)
- or continuous (space and time)
- Types of ordered points: Crystals, quasicrystals, polycrystals, quasipolycrystals,...

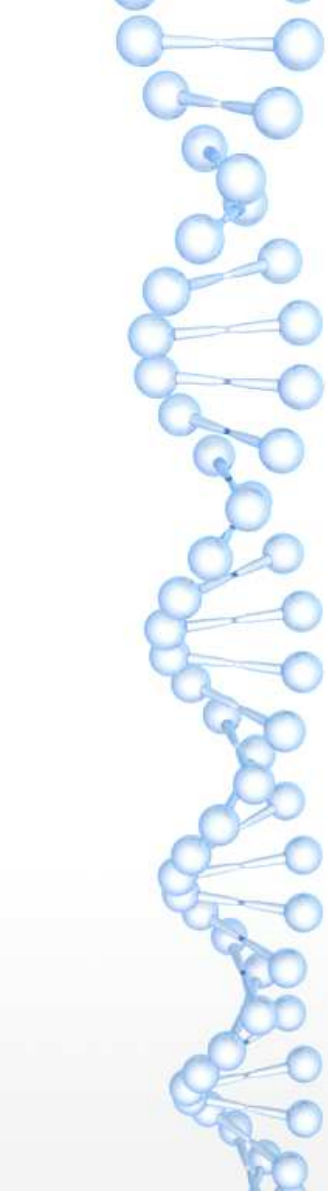
- Lines, surfaces, volumes, …Connected?
- a) Unoriented discrete
- b) Oriented discretes
- c) Continuous + oriented
- Graphs, digraphs, multidigraphs

Dimensions and hyperspaces (III)









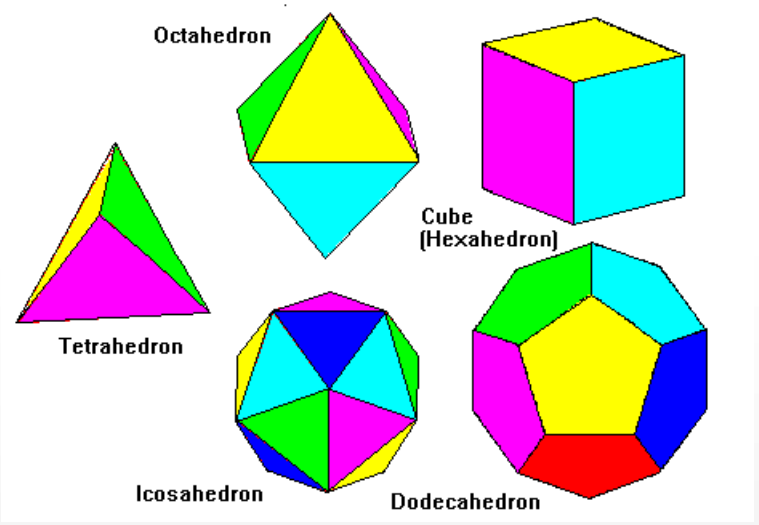
Vocabulary

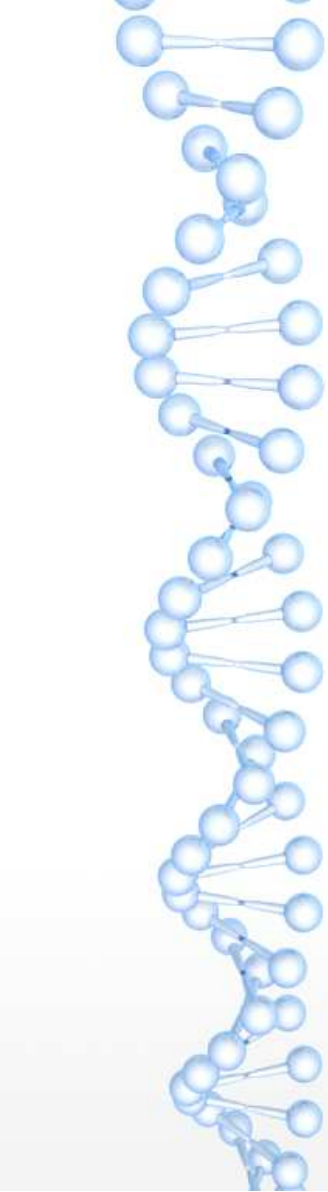
- Point to point: multigraphs, multidigraphs, apeiromultidigraphs.
- Polygons, polyhedra, polytopes (apeirogons, apeirohedra, apeirotopes,…). Others: amplituhedra, associahedra, the grassmannian, positroids, ...

- Special types: regular polygons and polytopes.
- D=2→ Infinite regulare polygons, D=3→ 1+4 regular polytopes, D=4 → 6 regular polytopes, D>4 → 3 regular polytopes.

D=2 and D=3 regular polygons and polyhedra

- D=2: equilateral triangle, square, regular pentagon, …,regular n-gon.
- D=3: tetrahedron (simplex), cube/hexahedron, octahedron, dodecahedron, icosahedron.
- The platonic
- polyhedra (3d):



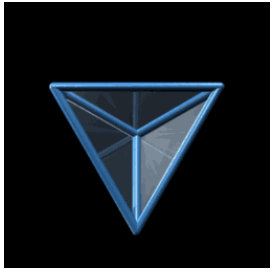


D=4 polytopes!

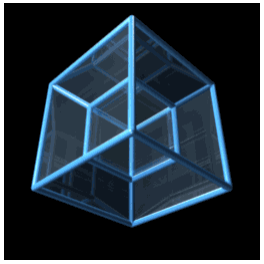
- #1. 5-cell, 4-simplex, pentatope, pentahedroid, pentachoron. C-5
- #2. C-8, 8-cell, octachoron, octahedroid, tetracube, tesseract, hypercube.
- #3. C-16, hexadecahedroid, hexadecachoron,orthoplex, 16-cell.
- #4. C-24, 24-cell, icositetrachoron, octaplex, hyperdiamond, polyoctahedron.
- #5. C-120, 120-cell, dodecaplex, hyperdodecahedron, polydodecahedron, hecatonicosachoron, dodecacontachoron and hecatonicosahedroid.
- #6. C-600. hexacosichoron and hexacosihedroid.

D=4 polytopes (II)!

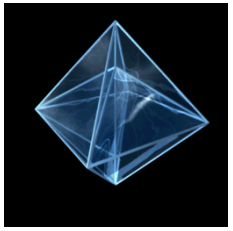
- 5-cell



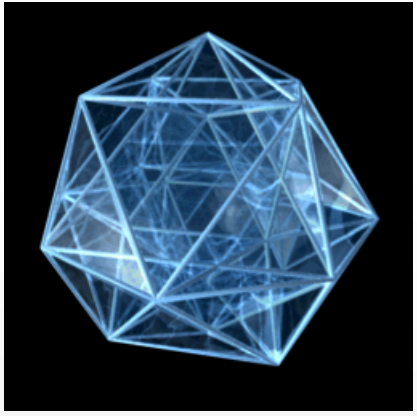
- 8-cell



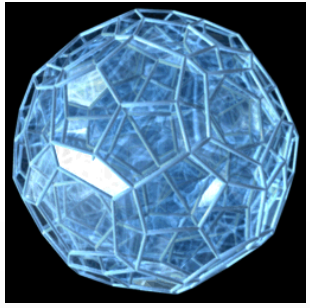
- 16-cell

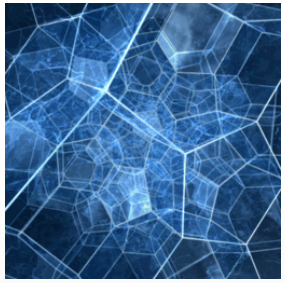


- 24-cell

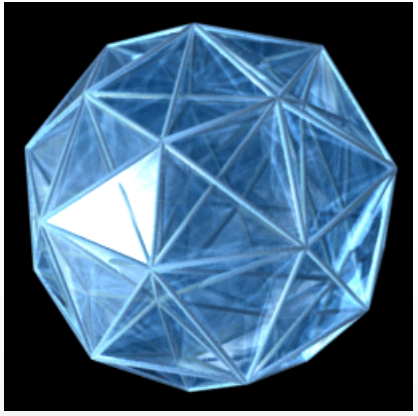


- 120-cell





- 600-cell



D>4. Stranger things happen...

- Only 3 (+1 exotica) regular polytopes.
- #1. n-simplex (n>4). Hypertetrahedron (5-cell=4-simplex)
- #2. cross-polytope, n-orthoplex, hyperoctahedron, cocube.
- #3. n-cube, hypercube, hypertesseract, n-dimensional cube (non-regular→ n-orthotope).
- Oddity: n-pentagonal polytope (non-regular uniform).

- Curiosities: n-cube has 2^n points, 2n-sides or faces.
- E(m,n), m<n, m-cubes in the boundary of n-cubes: $2^{n-m}\binom{n}{m}$
- Formulae for the n-cube:
- $V = L^n$ $S = (2n)L^{n-1}$

D > 4. More formulae

- N-orthoplex

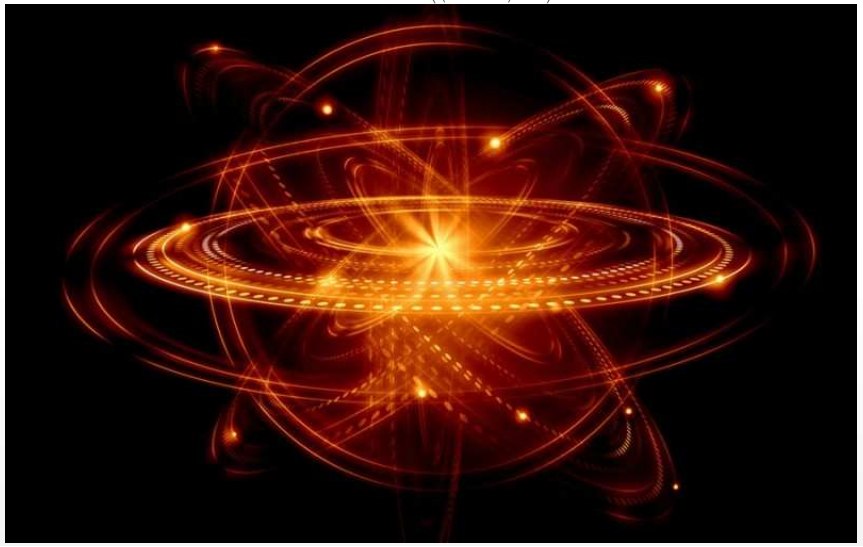
$$V=\frac{\sqrt{2^n}}{n!}L^n \qquad S=\frac{\sqrt{2^{n+1}n}}{(n-1)!}L^{n-1} \qquad E(k,n)=2^{k+1}\binom{n}{k+1}$$

- N-simplex

$$V=\frac{\sqrt{2^{-n}(n+1)}}{n!}L^n \qquad S=\frac{\sqrt{2^{1-n}n}}{(n-1)!}(n+1)L^{n-1} \qquad E(k,n)=\binom{n+1}{k+1}$$

- Not a polytope, but the hypersphere volume and hypersurface

$$V=\frac{\Gamma(1/2)^n R^n}{\Gamma(n/2+1)} \qquad S(n-1)=dV(n)/dR \rightarrow S(n)=\frac{2\Gamma(1/2)^{n+1}}{\Gamma((n+1)/2)} \qquad \Gamma(n)=(n-1)!\Leftrightarrow\Gamma(1/2)=\sqrt{\pi}$$



Conclusion (Waku, waku!)

Explore higher dimensions and multiverses!
(Never stop exploring!)