Teaching Physics with Physmatics
Enseñando Física con Fismática

J. F. G. H.[1]

[1]Space-time Foundation, Eccentric Multiverse of Madness
Quantum TimeLord Virtual Academy

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XXXVIII Biennial of Physics of the Spanish Royal Physics Society (R.S.E.F.) (Murcia, 11-15 July 2022)
XXXVIII Reunión Bienal de la Real Sociedad Española de Física (Murcia, del 11 al 15 de julio de 2022), Earth planet
Milky Way Galaxy, Laniakea, Known Universe (The Multiverse)

Contenido

What Physmatics IS/Lo que es la Fismática

Gráfica de lo que es la Ciencia de la Fismática en comparación a otros enfoques/Plot of what Physmatics is compared to other approaches...

No Physics $\longleftrightarrow$ Higher Physics

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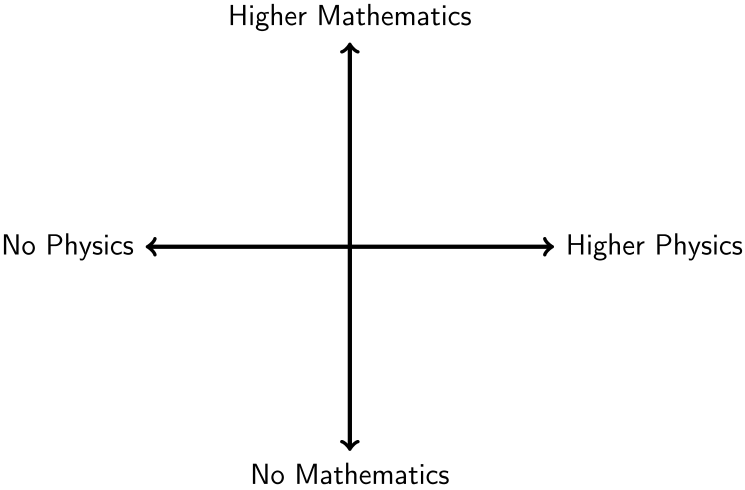

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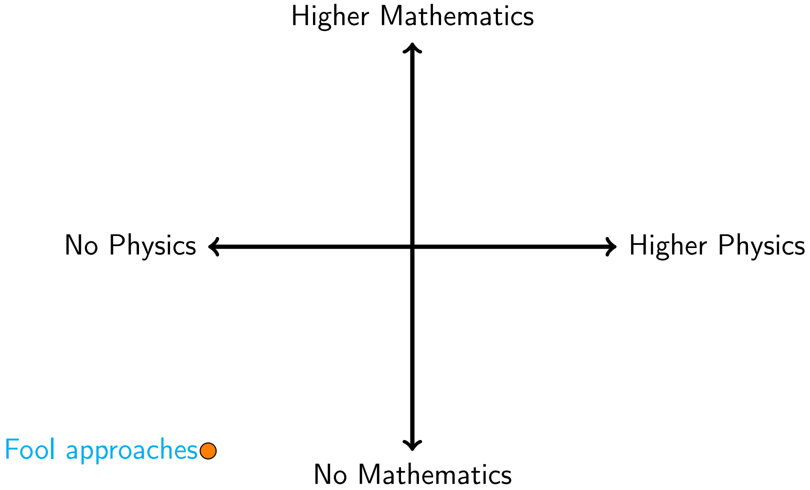

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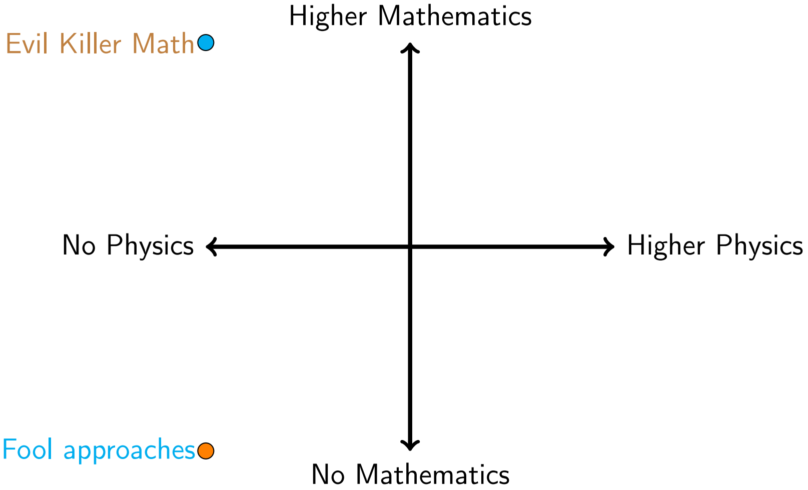

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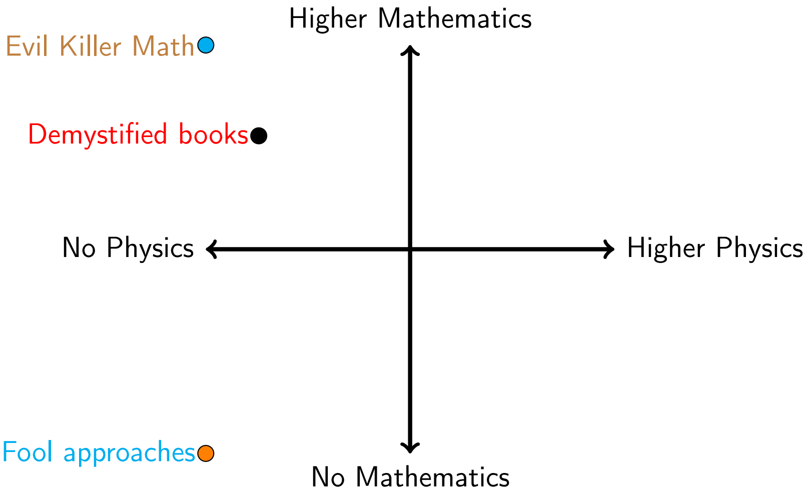

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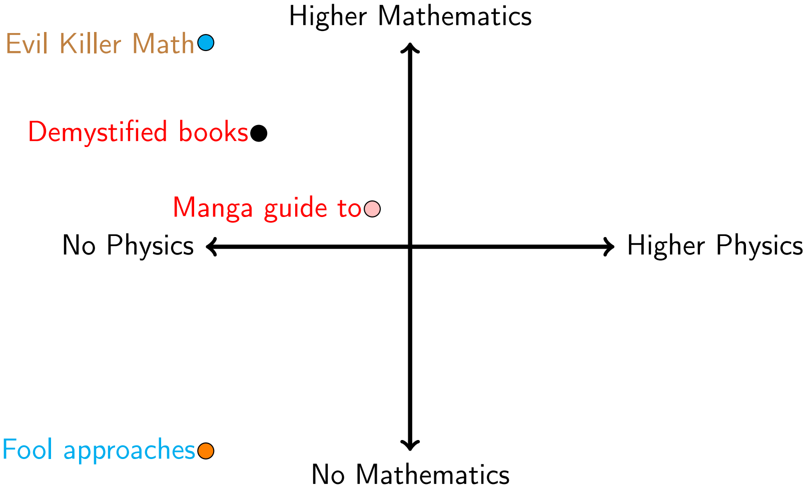

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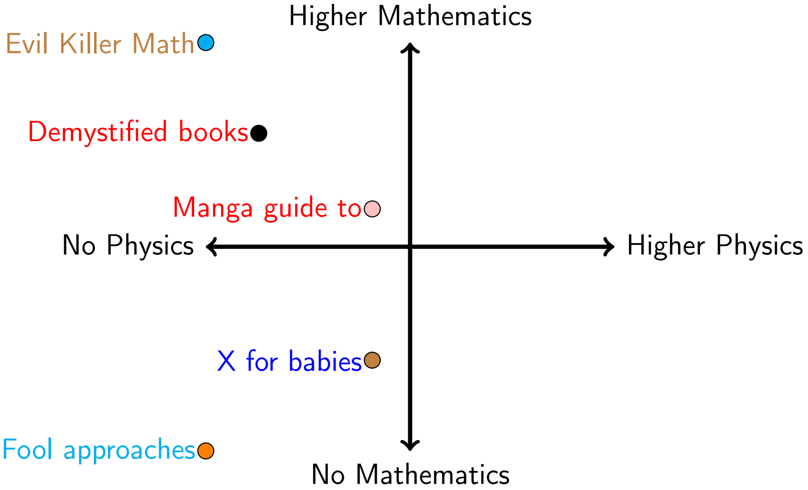

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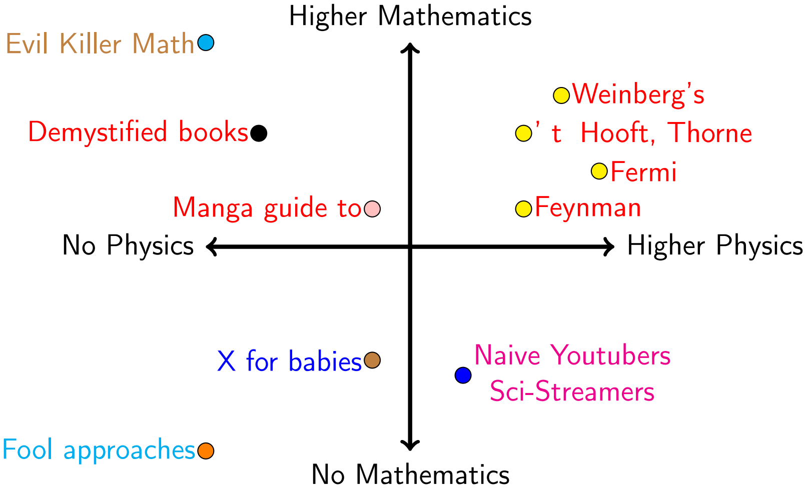

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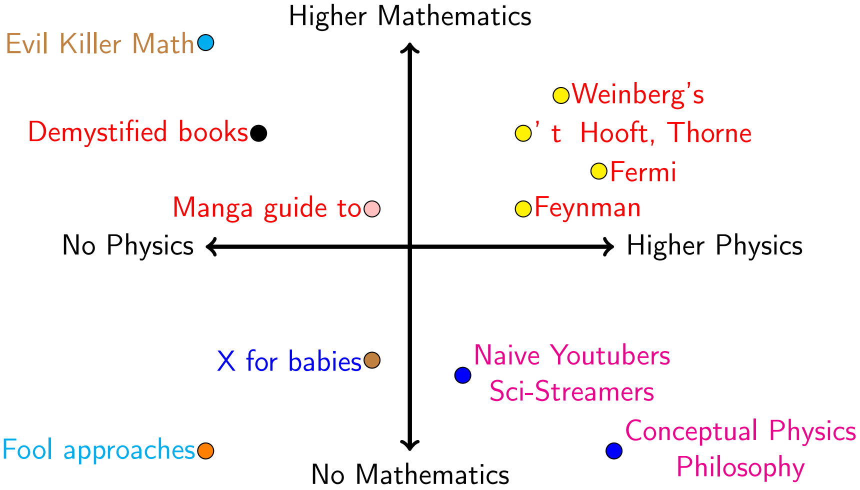

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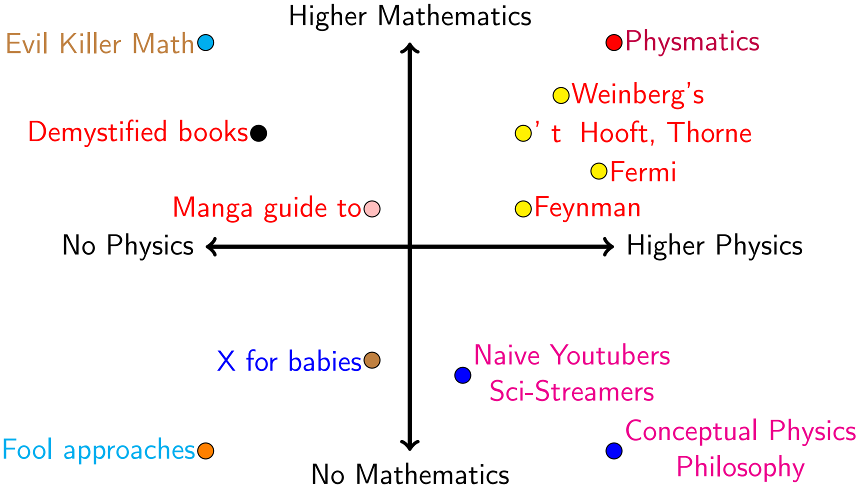

Contenido

The hypersphere(I)/la hiperesfera(I)

Hypersphere

Hypersphere (euclidean) or n-sphere is the geometrical locus or manifold S^n with (n-1)-dimensional constraint $\sum_{i=1}^n x_i^2 = x_1^2 + \cdots + x_n^2 = R^2$.

Hypervolume and hypersurface for n-spheres

The hypervolume $V(S^n)$ and hypersurface $\Sigma(S^n)$ is calculated as follows:

$$V_n = \frac{\Gamma(1/2)^n R^n}{\Gamma\left(\frac{n}{2}+1\right)} \quad \Sigma_{n-1} = \frac{dV_n}{dR} = \frac{n\Gamma^n(1/2)R^{n-1}}{\Gamma\left(\frac{n}{2}+1\right)} \tag{1}$$

where $\Gamma(1/2) = \sqrt{\pi} = (-1/2)!$. Remark: $V(S^\infty) = 0$, and the volume of the 23-sphere unit sphere is equal to the Leech lattice volume $\Lambda_{24} = \pi^{12}/12!$ behind the symmetry of the monster group M. Dimensional recurrence: $V_n = \dfrac{R\Sigma_{n-1}}{n}$.

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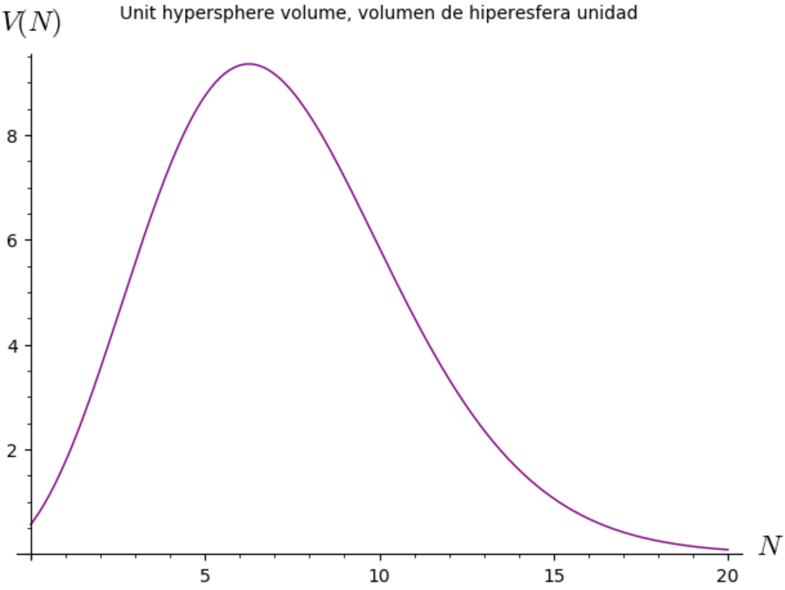
Gráfica/Graphics



Solve, exactly, the following equations/Resolver, exactamente, las siguientes ecuaciones:

Special functions/Funciones especiales

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- $xe^x = z$
- $xa^x = Y$
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- $\ln(A + BX) + CX = \ln(D)$
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Hint: Use Lambert W-function! Should we teach it at (high)-school/University?
Pista: Usar la función W de Lambert. ¿Deberíamos enseñarla en la Universidad o en el IES?

Deformed calculus/Cálculo deformado

Everyone knows what a derivative is...

$$Df(x) = \frac{f(x+h) - f(x)}{h} \tag{3}$$

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Moreover, take the fractional Riemann-Liouville derivative

$${}_aD_x^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x f(t)(x-t)^{\alpha-1}\, dt. \tag{5}$$

There are more, but...

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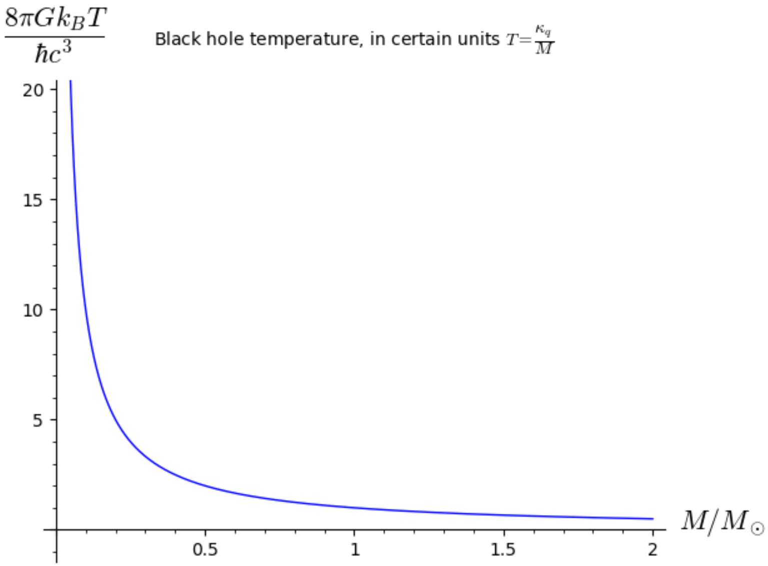
Proportion and dimensional analysis

- A tool for Black hole chemistry? ¿Un instrumento para la Química de agujeros negros?
- Via power laws/Via leyes de potencias...

Proportion and dimensional analysis



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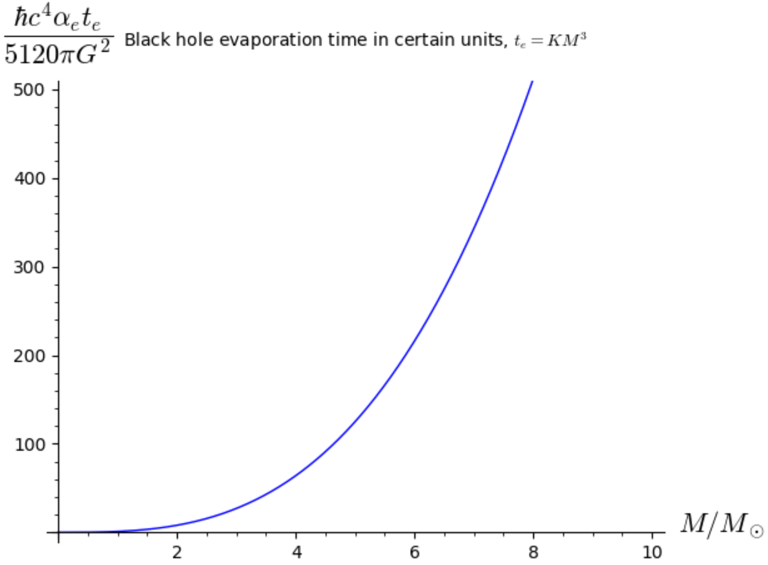

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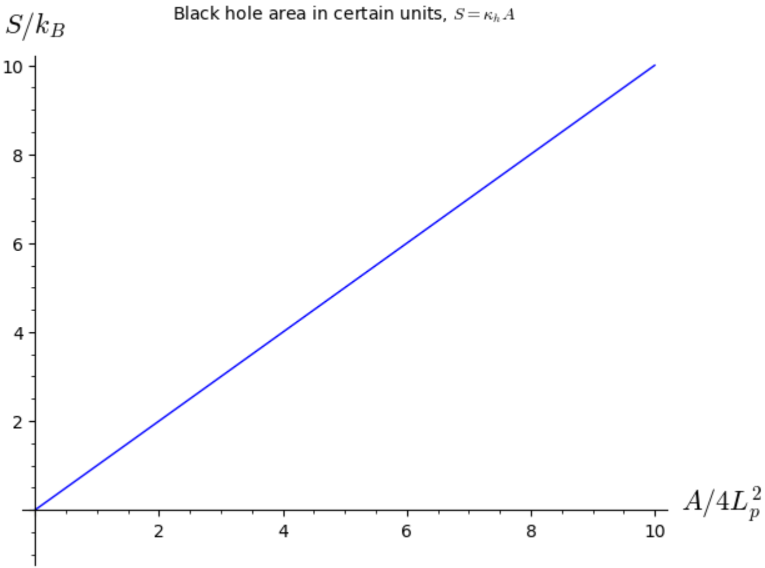

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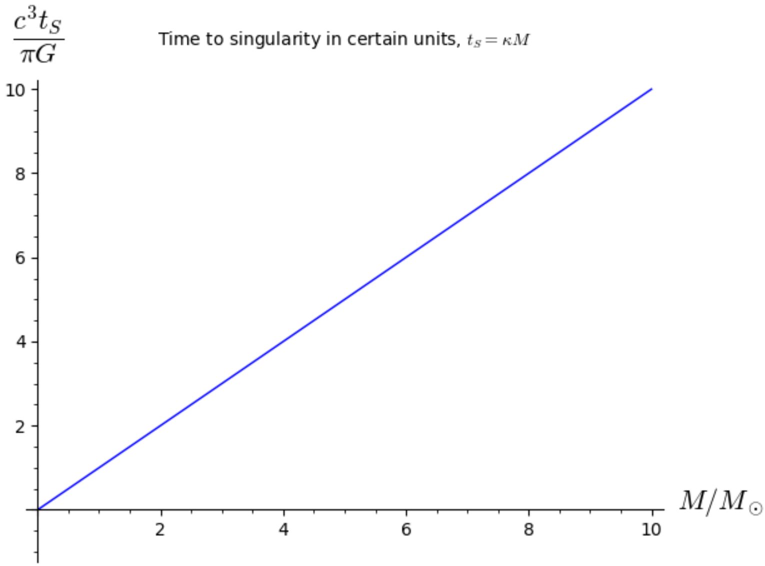

Kepler 3rd law variants(I)

Usual Kepler 3rd law: $T^2 = \dfrac{4\pi^2}{GM_B}R^3$. What about some variants? For Kerr black holes:

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$$\Omega = \pm \frac{M^{1/2}}{r^{3/2} \pm aM^{1/2}} \tag{6}$$

where $M = G_N m$ is the mass in gravitational natural units, a is the Kerr rotation parameter $a = cJ/M = J/Mc$. In terms of complete dimensional constants reads

$$\Omega = \pm \frac{\sqrt{G_N M}}{r^{3/2} \pm \chi \left(\frac{\sqrt{G_N M}}{c}\right)^3} = \pm \frac{c^3}{GM}\left(\pm\chi + \left(\frac{c^2 r}{G_N M}\right)^{3/2}\right)^{-1} \tag{7}$$

Kepler 3rd law variants(II): beyond standard gravity

For gravitational theories with effective potential:

$$V_e = -\frac{GM}{r}\left(1 + A\frac{M^p}{r^p}\right) + \frac{L^2}{2\mu^2 r^2}$$

circular orbit condition reads $V' = 0$, and with $L = \mu r^2 \Omega^2$ you get the generalized Kepler 3rd law

$$\Omega^2 = \frac{GM}{r^3}\left(1 + \frac{M^p A(p+1)}{r^p}\right) \tag{8}$$

More? Take the Finslerian-like 3rd law modification:

$$\frac{r^3}{T^2} = \left(1 - \frac{A(r)}{r^4}\right)\frac{GM}{4\pi^2} \tag{9}$$

Kepler 3rd law variants(II): strong gravity modified QG?

Recently, it has been proposed a modified gravitational law with effective potential energy:

$$U_e = -\frac{GMm}{r} - \lambda Mm \ln\left(\frac{r}{r_0}\right)$$

giving

$$F(r) = -\frac{GMm}{r^2} - \frac{\lambda Mm}{r} = -G_e\frac{Mm}{r^2} \tag{10}$$

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Similar ideas are proposed by asymptotically safe gravity approaches(by Weinberg and others), where $G_e = G(r)$, or even superstrings with $G = G(r,t) = g_s^2 L_s^2 e^{\phi(r,t)}$.

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Quantum gravity, string theory and molecular forces

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$$E_p = U = -\frac{GM_1M_2}{r}\left(1 + a\frac{G(M_1+M_2)}{c^2 r} + b\frac{G\hbar}{c^3 r^2}\right) \tag{12}$$

with $a = 3$ (GR) and $b = 41/10\pi$ (QG at one loop). Reciprocally, we could take the effective force in 26d bosonic string theory, namely

$$F_N = G_{26d}\frac{Mm}{r^{24}} \tag{13}$$

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Exercise: calculate U_{26d}!. It can be shown $G_{26d} = g_s^2 L_s^{24}$ in certain units. Intermolecular force between diatomic molecules can be approximated by the central force:

$$f(r) = -\frac{K_1}{r^6} + \frac{K_2}{r^{12}}$$

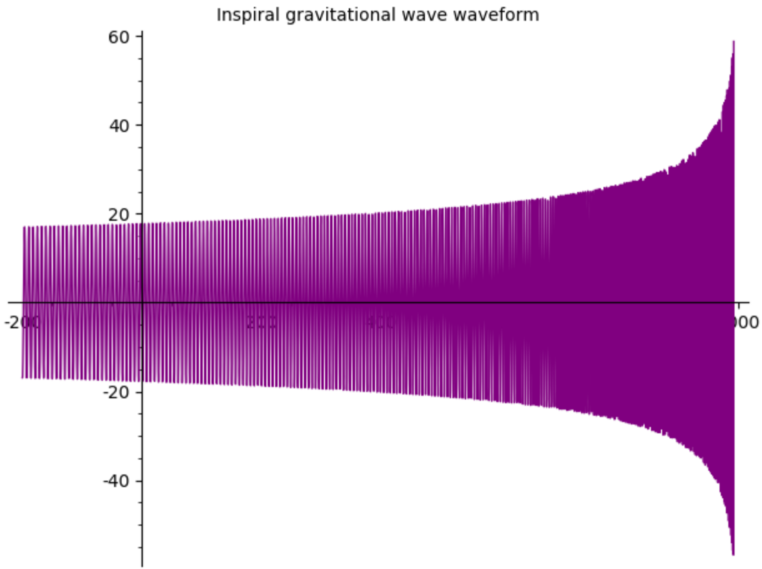
Find out the potential energy for this force.

Certain oscillating system moves with waveform, S.I. units,:

$$\Psi = 100(1000 - t)^{-1/4} \cos\left((10(1000 - t)^{5/8}\right)$$

a) Is this a SHO? Explain the answer.
b) Plot $y(t)$.
c) Calculate the vibrational speed and acceleration, $v = d\Psi/dt$, $a = d^2\Psi/dt^2$.
d) Calculate when is maximum the vibrational speed and its value, and the value of Ψ at that value. Comment the results.

Waves(I): inspiral GW



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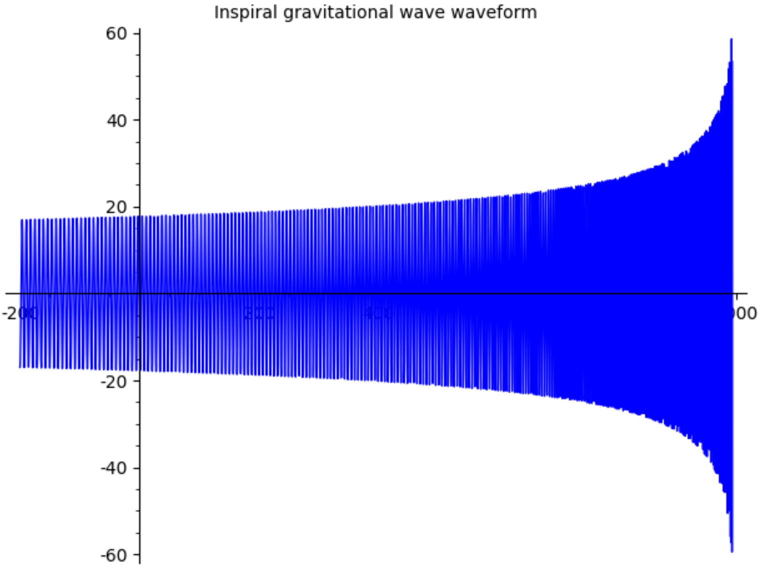

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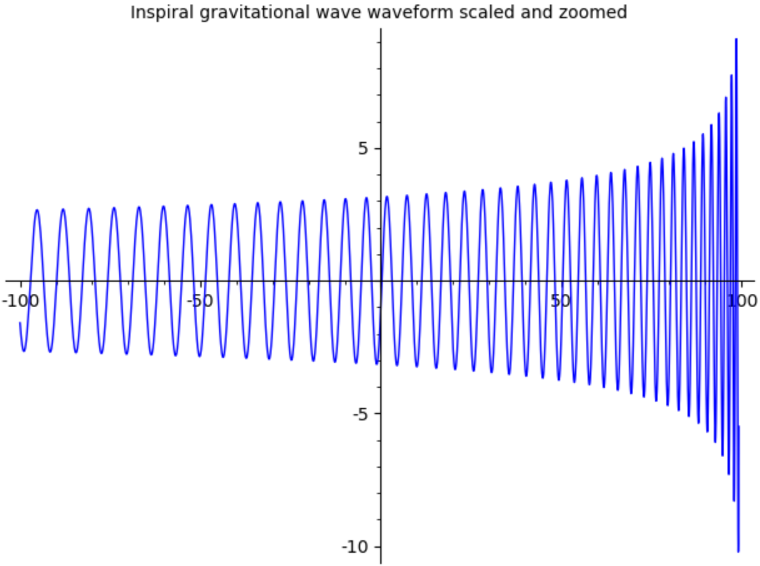

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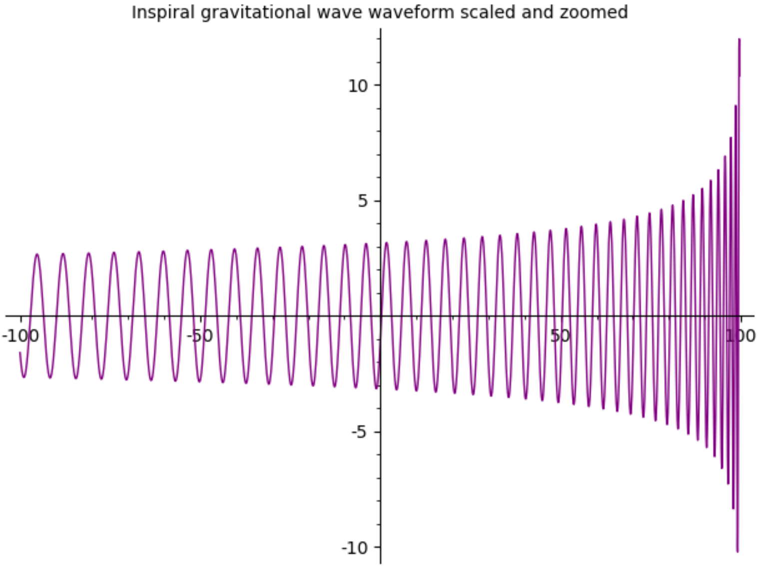

Waves(II): Axion DM model

A model for dark matter [5], which density reads $\rho_{DM} = 0{,}4 GeV/cm^3$, predicts a distribution with waveform due to ultralight (pseudo)scalar particles. Scalar waves along positive OX are given by:

$$\phi(x,t) = 2\frac{\sqrt{2\rho_{DM}}}{m_\phi c^2}\cos\left(2\pi\frac{m_\phi c x}{h}\right)\sin\left(2\pi\frac{m_\phi c^2 t}{h}\right)$$

with $c = 3\cdot 10^8 m/s$ is the speed of light, $h = 6{,}63\cdot 10^{-34} J\cdot s$ is the Planck constant, m_ϕ and is the mass of particles ϕ. Determine, if $E_\phi = m_\phi c^2 = 10\cdot 10^{-22} eV = 1 zeV$:

a) Type of wave and explanation, the propagation speed v_p, wavevelength λ_ϕ, frequency f_ϕ and period T_ϕ. (0.5 points=0.1x5).

b) Amplitudes for ϕ and ϕ^2 with S.I. units (0.5 points).

Waves(II): Axion DM model

c) The number of maxima we would expect to see in the square of particle distribution $\phi(0,t)^2$, using a haloscopic detector, in a time equal to the period, $t = T_\phi$. (0.5 points)
d) Number of particles per cubic meter if all the dark matter is locally made of ϕ particles. What happens to the field and the wavefunction in the limit of massless field, i.e. $m_\phi = 0$? (0.5 points)
Data: $e = 1{,}6 \cdot 10^{-19} C$

Cosmography(I)

In Cosmography we define, for $a(t)$:

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$$\boxed{\text{Hubble function}: \quad H(t) = \frac{1}{a}\frac{da}{dt}} \tag{14}$$

$$\boxed{\text{deceleration function}: \quad q(t) = -\frac{1}{aH^2}\frac{d^2a}{dt^2}} \tag{15}$$

$$\boxed{\text{jerk}: \quad j(t) = \frac{1}{aH^3}\frac{d^3a}{dt^3}} \quad \boxed{\text{snap}: \quad s(t) = \frac{1}{aH^4}\frac{d^4a}{dt^4}} \tag{16}$$

$$\boxed{\text{crackle/lerk function}: \quad l(t) = \frac{1}{aH^5}\frac{d^5a}{dt^5}} \tag{17}$$

Cosmography(II)

Cosmographic parameters in current time are denoted by $(H_0, q_0, j_0, s_0, l_0)$ and are a target of present and future cosmological measurements. Since:

$$a(t) = a(0) + \frac{da}{dt}(0)t + \frac{d^2a}{dt^2}(0)t^2 + \cdots + \frac{1}{n!}\frac{d^na}{dt^n}(0)t^n + \mathcal{O}(t^{n+1}) \quad (18)$$

and then H with redshift z

$$H(z) = H(0) + \frac{dH}{dz}(0)z + \frac{d^2H}{dz^2}(0)z^2 + \cdots + \frac{1}{n!}\frac{d^nH}{dt^n}(0)z^n + \mathcal{O}(z^{n+1}) \quad (19)$$

a) Check, with explicit calculations, the relationships below :

$$\dot{H} = -H^2(1+q)$$

$$\ddot{H} = H^3(3q + j + 2), \quad \dddot{H} = H^4(-3q^2 - 12q - 4j + s - 6)$$
$$\ddddot{H} = H^5(30q^2 + 60q + 10qj + 20j - 5s + l + 24)$$

b) Find out the dimensions and units of cosmographic parameters, explaining why the Hubble law is related to them, $v = H(z)d(z)$.

Cyclic model of the universe

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$$\ddot{a} + \frac{|\Lambda|}{3} a = \frac{4\pi G_N c}{3}, \text{ with solution } a(t) = \frac{1}{\gamma\omega}\left(1 + \sqrt{1-\gamma}\cos(\omega t)\right) \tag{20}$$

where the frequency and width of oscillation are defined as:

$$\omega = \sqrt{\frac{|\Lambda|}{3}}, \quad \gamma = \frac{3\,|\Lambda|}{(4\pi G_N c)^2} \sim \frac{a_-}{a_+} \tag{21}$$

We used the Friedmann equations for FRW and the state equation $p = \omega\rho$, with $\rho = -2/3$ (domain walls).

Inflation and the Multiverse (I)

How many Universes in the Multiverse? [3], i.e., ¿cuántos universos hay en el Multiverso? Respuestas posibles (A.Linde et al.; cf. ideas de A. Vilenkin y A. Guth):

- Slow-roll inflation provides $\mathcal{N}_{efolds} \sim e^{e^{3N}}$. If $N = 60$ (usual hypothesis), then $\mathcal{N} \sim e^{e^{180}} \sim 10^{10^{77}} \sim 10^{S_{BH}(1M_\odot)}$.
- (Chaotic) Eternal inflation provides, $N = cS_{dS} \sim c/m$: $\mathcal{N} \sim e^{e^{3N}} \sim e^{e^{3c/m}} \sim 10^{10^{10^7}} >> 10^{\text{googol}}$
- Cosmological constant universes, Λ non-zero, imply $S_p \approx H^{\overline{\frac{1+3\omega}{1+\omega}}} |\Lambda|^{-\overline{\frac{1+3\omega}{2+2\omega}}} = H^{3/2} |\Lambda|^{-3/4} \to \mathcal{N} \sim 10^{10^{82}}$.
- Universes in the string landscape (M is the number of dS vacua):

$$\mathcal{N} \approx \sum_{j=1}^{M} \exp\left(|\Lambda|^{-3/4}\right) \sim e^{0{,}75M}, \quad \text{Popularly } M \sim 10^{500}, \mathcal{N} \sim 10^{10^{375}}$$

  Other options: $M \sim 10^{272000}$ (F-theory), $M \sim 10^{15}$. $\Lambda \sim 10^{-122}$, $N_{max} \sim log|\Lambda| \sim 290$, and $N(efolds) \sim 70$.

Inflation and the Multiverse (II)

- Our (observable) Universe has an entropy bound $S_{dS} \leq |\Lambda|^{-3/4} \sim 10^{90}$.
- Milky way black hole entropy: $S_{MWBH} \sim 10^{100} \sim$ googol.
- The number of observers with masses about $M \sim 10^2 kg$, and height $1m$ is bound by Bekenstein formula

$$\mathcal{N}_{obs} \leq S_{Bek} = e^{2\pi MR} \leq e^{10^{45}}$$

Remarkly, the number of an intelligent brain observer is about $\mathcal{N} \sim 10^{10^{16}} >> \mathcal{N}_{ds-Vacua}$, a brain seems to have more configurations than the expected possible geometries of the Universe(Multiverse).

Inflation and the Multiverse (II)

- Our (observable) Universe has an entropy bound $S_{dS} \leq |\Lambda|^{-3/4} \sim 10^{90}$.
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Remarkly, the number of an intelligent brain observer is about $\mathcal{N} \sim 10^{10^{16}} >> \mathcal{N}_{ds-Vacua}$, a brain seems to have more configurations than the expected possible geometries of the Universe(Multiverse).

En la inflación, un campo escalar en modelos de inflación cáotica eterna $V(\phi) = 0{,}5 m^2\phi^2$ tiene un comportamiento de MAS modulado en amplitud

$$\phi(t) = \Phi(t) \cdot \sin(m\phi), \;\; \text{with} \;\; \Phi(t) = \frac{M_p}{\sqrt{3\pi} mt} \sim \frac{M_p}{2\pi\sqrt{3\pi} N} \tag{22}$$

and N is the number of oscillations since the end of inflation!

Inflation energy scale and tensor-to-scalar ratio

General inflation models use ODE like the Lame or Mathieu equations (H is a friction term), or elliptic functions. Moreover, the energy scale of inflation is related to the tensor-to-scalar perturbation ratio.

Inflation energy scale

$$V^{1/4} \approx \left(\frac{3\pi^2}{2} r\mathcal{P}_s\right)^{1/4} M_P = \left(\frac{r}{0{,}01}\right)^{1/4} \cdot 1{,}06 \cdot 10^{16}\text{GeV} \qquad (23)$$

Primordial gravitational waves triggered by inflation in the Early Universe (even in the beginning of time or "before" the Big Bang) are a hot target of current and future research!

References for the last 2 slides:

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[2] *Constraining Feeble Neutrino Interactions with Ultralight Dark Matter*, arXiv: https://arxiv.org/abs/2205.06821v1 and references therein. Abhish Dev, G. Krnjaic, P. Machado, and H. Ramani.

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[8] *Mass and Motion in General Relativity*, Springer. Luc Blanchet, Alessandro Spallicci, Bernard Whiting.

I could do this every day of my life...

My passion is Physmatics...Do I ever sleep?

Please. . . I am grateful for your attention!

I know...Things just got out of hands...I apologize!

Gracias/Thank you!

Back-up slides

Objetos geométricos: la hiperesfera(II)

$$V(S^0) = 2R \tag{24}$$

$$V(S^1) = \pi R^2 \approx 3{,}14159 R^2 \tag{25}$$

$$V(S^2) = \frac{4}{3}\pi R^3 \approx 4{,}11879 \tag{26}$$

$$V(S^3) = \frac{\pi^2}{2} R^4 \approx 4{,}9348 R^4 \tag{27}$$

$$V(S^4) = \frac{8\pi^2}{15} R^5 \approx 5{,}26379 R^5 \tag{28}$$

The amazing vanishing sphere volume with increasing dimensions!!!!!!!!!!

Objetos geométricos: la hiperesfera(II)

$$V(S^5) = \frac{\pi^3}{6} R^6 \approx 5{,}16771 R^6 \tag{24}$$

$$V(S^6) = \frac{16\pi^3}{105} R^7 \approx 4{,}72477 R^7 \tag{25}$$

$$V(S^7) = \frac{\pi^4}{24} R^8 \approx 4{,}05871 R^8 \tag{26}$$

$$V(S^8) = \frac{32\pi^4}{945} R^9 \approx 3{,}29851 R^9 \tag{27}$$

$$V(S^9) = \frac{\pi^5}{120} R^{10} \approx 2{,}55016 R^{10} \tag{28}$$

The amazing vanishing sphere volume with increasing dimensions!!!!!!!!!!

Objetos geométricos: la hiperesfera(II)

$$V(S^{10}) = \frac{64\pi^5}{10395} R^{11} \approx 1{,}8841 R^{11} \tag{24}$$

$$V(S^{11}) = \frac{\pi^6}{720} R^{12} \approx 1{,}33526 R^{12} \tag{25}$$

$$V(S^{12}) = \frac{128\pi^6}{135135} R^{13} \approx 0{,}919629 R^{13} \tag{26}$$

$$V(S^{13}) = \frac{\pi^7}{5040} R^{14} \approx 0{,}599265 R^{14} \tag{27}$$

$$V(S^{14}) = \frac{256\pi^7}{2027025} R^{15} \approx 0{,}381443 R^{15} \tag{28}$$

The amazing vanishing sphere volume with increasing dimensions!!!!!!!!!!

Objetos geométricos: la hiperesfera(II)

$$V(S^{15}) = \frac{\pi^8}{40320} R^{16} \approx 0{,}235331 R^{16} \tag{24}$$

$$V(S^{16}) = \frac{512\pi^8}{34459425} R^{17} \approx 0{,}140981 R^{17} \tag{25}$$

$$V(S^{23}) = \frac{\pi^{12}}{479001600} R^{24} \approx 0{,}00192957 R^{24} \tag{26}$$

$$V(S^{24}) = \frac{8192\pi^{12}}{7905853580625} R^{25} \approx 0{,}000957722 R^{25} \tag{27}$$

$$V(S^{25}) = \frac{\pi^{13}}{6227020800} R^{26} \approx 0{,}000466303 R^{26} \tag{28}$$

The amazing vanishing sphere volume with increasing dimensions!!!!!!!!!!

$$V(S^{26}) = \frac{16384\pi^{13}}{213458046676875} R^{27} \approx 0{,}000222872 R^{27} \tag{24}$$

$$V_{91} \begin{cases} \dfrac{\pi^{46} R^{92}}{55026221598120889498503054288002548929616517529600000000000} \\ \approx 1{,}34377 \cdot 10^{-35} R^{92} \end{cases} \tag{25}$$

Two more...The 4096-dimensional sphere

$$V(S^{4095}) \approx 8{,}70008138919055 \times 10^{-4877} R^{4096} \tag{26}$$

with a fantastic fraction that can not be written in the margin or space of this page easily. Surprisingly, the infinite-dimensional sphere volume is zero: The amazing vanishing sphere volume with increasing dimensions!!!!!!!!!!

Objetos geométricos: la hiperesfera(II)

$$V(S^\infty) = 0 \tag{24}$$

The amazing vanishing sphere volume with increasing dimensions!!!!!!!!!!

Bohr-logy in XD

2d Bohr energy levels and radius

In any 2d Universe, the Bohr-Rydberg energy and radius are:

$$E = Ke^2\left(\frac{1}{2}+\ln(n)\right), \quad r_n = na_0(2d) = \frac{n\hbar}{\sqrt{mKe}}, n \in \mathbb{Z} \tag{25}$$

For gravitational case, take $Ke^2 \rightarrow GMm$.

Dd Bohr energy levels and radius

In any Dd Universe, the Bohr-Rydberg energy and radius are:

$$E = \frac{D-4}{2(D-2)}\left(\frac{m}{\hbar^2}\right)^{\frac{D-2}{4-D}} n^{-\frac{2D-4}{4-D}} e^{\frac{4}{4-D}}, \quad r_n(D) = \left(\frac{m}{\hbar^2}\right)^{\frac{1}{D-4}} e^{\frac{2(2-D)}{(4-D)(D-2)}} n^{\frac{2}{4-D}} \tag{26}$$

For gravitational case, take $Ke^2 \rightarrow GMm$.

Gravity in XD

Newton in higher dimensions

In any Dd ($D = d+1$) Universe (spacetime), the gravitational force, the gravitational field, the potential energy and the potential read

$$F_N = G_D\frac{Mm}{r^{D-2}} = G_{d+1}\frac{Mm}{r^{d-1}} \quad g = G_D\frac{M}{r^{D-2}} = G_{d+1}\frac{M}{r^{d-1}} \tag{27}$$

$$U_g = G_D\frac{Mm}{r^{D-3}} = G_{d+1}\frac{Mm}{r^{d-2}}$$

$$\mathrm{V}_g = G_D\frac{2\Gamma((D-1)/2)M}{\pi^{(D-3)/2}r^{D-3}} = G_{d+1}\frac{2\Gamma(d/2)M}{\pi^{(d-2)/2}(d-2)r^{d-2}} \tag{28}$$

Dilution of gravity: $G_N(4d) = G_D/V_D$. $g^2_{YM}(4d) = g^2_{YM,d}R^{-d}$,

$M_P = \sqrt{hc/G} \sim 10^{-5}g, M_W = \frac{h}{c}\sqrt{\Lambda/3} \sim 10^{-65}g. Gh\Lambda/c^3 \sim 10^{-121}$.

$M_U = \frac{c^2}{G}\sqrt{3/\Lambda} \sim 10^{56}g,\ M'_W = \sqrt[3]{\frac{h^2\sqrt{\Lambda/3}}{G}} \sim 10^{-25}g.\ M_U/M_W \sim 10^{121}$

Uniform sphere total energy in XD

Gravitational/electric energy for uniform density sphere

$$U_g = -G_{d+1}\frac{d(d-2)M^2}{d+2}\frac{1}{R^{d-2}} = -G_D\frac{(D-1)(D-3)M^2}{D+1}\frac{1}{R^{D-3}} \quad (29)$$

with $D = d+1$ and M the mass. If $M = \rho V$, then

$$U_g = -G_{d+1}\frac{d(d-2)\pi^d\rho^2}{(d+2)\Gamma^2\left(\frac{d}{2}+1\right)}R^{d+2} = -G_D\frac{(D-1)(D-3)\pi^d\rho^2}{(D+1)\Gamma^2\left(\frac{D+1}{2}+1\right)}R^{D+1} \quad (30)$$

Trickery for the electric case: substitute $G_n \to K_C$, $M \to Q$, with $Q = \rho V$.

Entropic gravity in XD

Hypothesis for $D = d + 1$ hyperdimensional Newton gravity:

- $A(\Sigma) = \dfrac{2\pi^{d/2}R^{d-1}}{\Gamma(d/2)}$.
- $N = A(\Sigma)/L_p^{d-1}$, $E = mc^2 = Nk_BT/2$, $\Delta S = 2\pi k_B \dfrac{mc\Delta x}{\hbar}$.

Then:

$$F = -T\frac{\Delta S}{\Delta x} = -G_d\frac{Mm}{R^{d-1}}$$

where

Hyperdimensional gravitational Newton constant

$$G_d = \frac{2\pi^{1-d/2}\Gamma\left(\dfrac{d}{2}\right)c^3L_p^{d-1}}{\hbar} = 2\pi^{1-d/2}\Gamma\left(\frac{d}{2}\right)\frac{c^3L_p^{d-1}}{\hbar}$$

$$\phi_g = -\Omega_d G_d M; \quad \phi_e = \Omega_d K_d Q = Q/\varepsilon_0(d) \;\; \Omega_d = 2\pi^{d/2}/\Gamma(d/2)$$

Take the functional equation:
$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\, \zeta(1-s)$ for $1 - s = d/2$. Then, since

$$G_d = 2\pi^{1-d/2}\Gamma\left(\frac{d}{2}\right)\frac{c^3 L_p^{d-1}}{\hbar}$$

you can derive that

Gravitational constant and zeta function

$$G_d = \frac{\pi 2^{d/2}\zeta\left(1-\frac{d}{2}\right)}{\zeta\left(\frac{d}{2}\right)\cos\left(\frac{\pi d}{4}\right)}\left(\frac{c^3 L_p^{d-1}}{\hbar}\right) \tag{31}$$

Classical atom instability

Hypothesis: non quantum atoms are unstable. To prove this:

$$P = \frac{dE}{dt} = \frac{2e^2a^2}{3c^3}(\text{Larmor formula})$$

$$\frac{Ke^2}{R^2} = \frac{mv^2}{R} \rightarrow v^2 = \frac{Ke^2}{mR},\ E = \frac{mv^2}{2} - \frac{Ke^2}{R} = -\frac{Ke^2}{R}$$

$$dt = -\frac{1}{\frac{dE}{dt}}\frac{dE}{dR}dR = -\frac{3}{16}\frac{m^2c^3R^2dR}{(E_0R_0)^2} \rightarrow \int_0^{t_c} dt = -\frac{3m^2c^3}{(E_0R_0)^2}\int_{R_0}^0 R^2dR$$

We finally get:

Decay time of classical em-atoms

$$t_c = \frac{m^2c^3R_0}{16E_0^2} = \frac{m^2c^3R_0^3}{4K_Ce^4} = \frac{4\pi^2\varepsilon_0^2m^2c^3R_0^3}{e^4} \simeq 1{,}6\cdot 10^{-11}s \sim 20ps$$

Gravitational music equations

Binary system with $M = M_1 + M_2$, $f_{GW} = 2f_{orb}$, $M_c = (M_1 M_2)^{3/5}/M^{1/5}$ yields (GR):

$$L_{GW} = \frac{2^5}{5}\left(\frac{G^{7/3}}{c^5}\right)\left[M_c \pi f_{GW}\right]^{10/3}$$

$$\dot{f}_{GW} = \left(\frac{96}{5}\right)\left(\frac{G^{5/3}}{c^5}\right)\left(\pi^{8/3}\right)\left(f_{GW}\right)^{11/3}$$

$$t_c = \frac{2}{2^8}\left(\frac{GM_c}{c^3}\right)^{-5/3}\left[\pi f_{GW}\right]^{-8/3}$$

Neutrino oscillations: the equations

Supposing transitions between different neutrino species, via $|\nu_\alpha\rangle$ to $|\nu_\beta\rangle$

Neutrino oscillations

$$\mathcal{A} = P(\alpha \to \beta) = |\langle \nu_\beta | \nu_\alpha \rangle|^2 = \left| \sum_j U^*_{\alpha j} U_{\beta j} e^{-i \frac{m_j^2 c^3 L}{2\hbar E}} \right|^2 \quad (32)$$

$$\mathcal{A} = \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\beta j} U^*_{\beta j}\right) \sin^2\left(\frac{\Delta m_{ij}^2 c^3 L}{4\hbar E}\right) +$$

$$+2 \sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\beta j} U^*_{\beta j}\right) \sin\left(\frac{\Delta m_{ij}^2 c^3 L}{2\hbar E}\right) \quad (33)$$ with

Neutrino transition matrix

$$|\nu_\beta\rangle = U^{\alpha}{}_{\beta} |\nu_\alpha\rangle$$

Multitemporal physics(I)

Usual 1T newtonian physics: $F = ma = m\frac{dv}{dt} = m\frac{d^2r}{dt^2} = -\nabla U(r)$, assuming conservative forces only. Let $W = F_i dx^i$ the work form, in a ND manifold $V \subset \mathbb{R}^N$, with submanifold nd $M \subset \mathbb{R}^n \subset \mathbb{R}^N$. $y^I = y^I(x)$, $\omega = f_I dy^I$ implies $dy^I = \frac{\partial y^I}{\partial x^i} dx^i$, and also

$$W = F_i(x)dx^i \rightarrow F_I = f_I(y(x))\frac{\partial y^I}{\partial x^i}$$

Single time manifold approach

$$f_I = m\delta_{IJ}\frac{d\dot{y}^J}{dt} = m\delta_{IJ}\frac{d^2y^I}{dt}$$

$$F_i = m\delta_{IJ}\frac{d\dot{y}^I}{dt}\frac{\partial y^J}{\partial x^i} = m\delta_{IJ}\frac{d^2y^I}{dt^2}\frac{\partial y^J}{\partial x^i}$$

Multitemporal physics(II)

Going multitemporal with timelike coordinates $(t) = t^\alpha$, $\alpha = 1, \cdots, m$

Multitime tensorial Newton 2nd law

$$f_I = m_{IJ}\delta^{\alpha\beta}\frac{\partial^2 y^J}{\partial t^\alpha \partial t^\beta}$$

$$f_i = m_{IJ}\delta^{\alpha\beta}\frac{\partial^2 y^I}{\partial t^\alpha \partial t^\beta}\frac{\partial y^J}{\partial x^i}$$

with anti-trace $F_i = F^\alpha_{i\alpha}$ given by the tensor 1-form

$$F^\sigma_{i\alpha} = m_{IJ}\delta^{\sigma\beta}\frac{\partial^2 y^I}{\partial t^\alpha \partial t^\beta}\frac{\partial y^J}{\partial x^i}$$

(Multitime) Kinetic energy

$$T = E_k = \frac{1}{2}m\delta_{IJ}\dot{y}^I\dot{y}^J \qquad T = \frac{1}{2}\delta_{IJ}\delta^{\alpha\beta}\frac{\partial y^I}{\partial t^\alpha}\frac{\partial y^J}{\partial t^\beta}$$

Multitemporal physics(III)

Single time Euler-Lagrange 1st order EOM

$$\delta S = 0 \rightarrow E(L) = \frac{\partial L}{\partial x^i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^i}\right) = 0$$

(Multitime) Euler-Lagrange EOM

$$\delta S = 0 \rightarrow E(L) = \frac{\partial L}{\partial x^i} - D_\alpha\left(\frac{\partial L}{\partial D_\alpha x^i}\right) = 0$$

(Multitime) Euler-Lagrange EOM: nth order

$$E(L) = \sum_{j=0}^{n}(-1)^j\left(\frac{\partial L}{\partial \partial_t^j x^i}\right) = 0 \quad E(L) = \sum_{J=0}^{n}(-1)^J\left(\frac{\partial^J L}{\partial D_\alpha^J x^i}\right) = 0$$

Multitemporal physics(IV)

Single time Hamilton EOM

Define 1T hamiltonian as $H = \dot{x}^i \dfrac{\partial L}{\partial \dot{x}^i} - L$, and $p_i = \partial L / \partial \dot{x}^i$, then

$$\dot{x}^i = \frac{dx^i}{dt} = \frac{\partial H}{\partial p_i} \qquad \dot{p}_i = \frac{dp_i}{dt} = -\frac{\partial H}{\partial x^i}$$

Multi-time Hamilton EOM

Define nT hamiltonian as $H = D_\alpha x^i \dfrac{\partial L}{\partial D_\alpha x^i} - L$, and $p_i^\alpha = \partial L / \partial D_\alpha x^i$, then

$$\frac{\partial x^i}{\partial t^\alpha} = \frac{\partial H}{\partial p_i^\alpha} \qquad \frac{\partial p_i^\beta}{\partial t^\alpha} = -\delta^\beta{}_\alpha \frac{\partial H}{\partial x^i}$$

Detecting exoplanets(I)

Astrometry

$$\theta = \left(\frac{M_p}{M_\star}\right)\left(\frac{a}{r}\right) \approx \frac{10^{-3}}{r(pc)}\left(\frac{P(yr)}{M_\star(\odot)}\right)^{2/3} M_p(J)$$

Here

$$V_r(m/s) \approx \frac{30}{(P(yr))^{1/3}} \frac{M_p(J)}{M_\star(\odot)^{2/3}} \sin(i)$$

Microlensing

$$R_E^2 = \frac{4GMD}{c^2}, \quad D = \frac{D_{ds}D_d}{D_s}, \quad t_0 = \frac{R_E}{v_e}$$

$$t_0 = \frac{2D_L\theta_E}{v_L} = \frac{2\theta_L}{v_L}\sqrt{\frac{4GM(1 - D_s/D_s)}{c^2 D_d}}$$

The impact parameter u reads

$$A = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}$$

Detecting exoplanets(III)

Direct detection

$$B \geq \frac{\lambda D}{r} \approx \left(\frac{\lambda}{10\mu mu}\right)\left(\frac{D}{10pc}\right)\left(\frac{r}{1AU}\right)^{-1} m$$

Detecting exoplanets(IV)

Radial velocity

$$K_\star = \left(\frac{2\pi G_N}{P}\right)^{1/3} \frac{M_p\,(M_\star + M_p)^{1/3}\sin(i)}{M_\star}\frac{1}{\sqrt{1-e^2}}$$

Also, it is usually written with $M_\star + M_P \simeq M_\star$ as follows

$$M_p \sin(i) = \left(\frac{P}{2\pi G}\right)^{1/3} K_\star M_\star^{2/3}\sqrt{1-e^2}$$

Bohr-like quantization of magnetic monopoles

Hypothesis:

- Magnetic and electric field of a point monopole charge with $Q_m = g$ and dual charge $e_g = eg/c = egv/c^2$.

$$F_e + F_m = 2F_{m,e} = F_c \leftrightarrow \frac{2K_c e_g}{R^2} = \frac{mv^2}{R} \rightarrow \frac{c^{-2}eg}{4\pi\varepsilon_0} = \frac{mvR}{2} = \frac{n\hbar}{2}$$

Then, $eg = \dfrac{n\hbar c^2}{2K_C}$(Q.E.D.). Equivalently: $\dfrac{g}{e} = \dfrac{nc}{2\alpha_e} \leftrightarrow \alpha_e = \dfrac{nce}{2g}$

- Dirac-Zwanziger-Schwinger dyonic quantization $Z = (e, g)$:

$$e_1 g_2 - e_2 g_1 = 2\pi n\hbar c$$

From this, it follows that $Q = ne$, or $Q = \left(n + \dfrac{1}{2}\right) e$ and

$$M = \sqrt{\frac{K_C}{G_N}} e = \frac{\hbar c^2}{g}\sqrt{\frac{1}{K_C G_N}}.$$

The existence of magnetic monopoles implies the quantization of Q_e.

Doctor Strange in the Multiverse of Madness!