Schwinger's effect: a short introduction

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Abstract

We review the Schwinger's mechanism and explain its theoretical and experimental importance. We expose shortly the current status of the experimental searches and revisit the main formulae, concepts and the right interpretation of this non-perturbative effect in 4d QED. The generalizations to lower and higher dimensional QED formulae are also provided and discussed in terms of polylogarithms.

1 Introduction

The Schwinger's mechanism or Schwinger effect is a non-perturbative QED phenomenon. It is the spontaneous production of e^+e^- pairs in the pressence of strong (usually constant) electric fields[1]. However, the Schwinger's mechanism is, yet, an untested or unseen, as far as we know, process in Quantum Electrodynamics[2](QED) in spite of the experimental effort that have been done to observe and measure it.

The production of electron and positron pairs by the electromagnetic field has a long story. Sauter [1] and Heisenberg et al. [2] had studied the process before the seminal paper by J. Schwinger [3], although they considered the problem in a less formal and more rude formalism, calculating the tunneling probability for the potential barrier generated by the electric field, a result that is qualitatively correct. Its original interpretation is not.

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[1]The concept, of course, can be generalized to general electromagnetic fields or even the electroweak field, the strong field and the gravitational field.

[2]Hence, it is a predicted unobserved SM physical phenomenon.

2 Schwinger's effect calculations

2.1 Heuristic calculation

There is a beautiful straight-hand derivation of the qualitative formula using elementary undergraduate physics. The work or energy to transport a charge q to a distance $2\lambda_c$ where λ_c is the Compton's wavelength is

$$Energy = W = qEd = qE2\lambda_c = 2E\frac{\hbar}{mc} \tag{1}$$

where we used $\lambda_c = \hbar/mv \approx \hbar/mc$ for $v \approx c$. Now, if we want to create a pair of particles, they have energy

$$E_{pair} = m_{e^+}c^2 + m_{e^-}c^2 = 2mc^2 \tag{2}$$

where we have supposed that $m_{e^+} = m_e^-$ by CPT invariance. We note that the observation of Schwinger's effect, if possible, could provide a novel way to test the identity between the particle and antiparticle masses, sice if we measure accurately and precisely the experimental energy of the created pair E^{exp}_{pair} we could measure the deviation

$$\delta E = E^{exp}_{pair} - E^{th}_{pair} = \epsilon mc^2 - 2mc^2 = (\epsilon - 2)mc^2 \tag{3}$$

$$\left|\frac{\delta E}{E^{th}_{pair}}\right| = \left|\frac{\epsilon}{2} - 1\right| \tag{4}$$

Now, to create the pair, we should have at least the pair energy, so

$$W = E_{pair} \to 2qE\frac{\hbar}{mc} \geq 2mc^2 \tag{5}$$

$$E \geq \frac{m^2c^3}{q\hbar} \tag{6}$$

$$Action = S \geq \frac{h}{2} = \frac{\pi m^2 c^3}{qE} \tag{7}$$

In Quantum Mechanics, using the path integral approach, the transition amplitude is essentially given by $Z = const.e^{-S}$ where S is the action, so then the probability for the Schwinger effect, taking $q = e$ will be

$$P \approx (Amplitude) \cdot e^{-\frac{\pi m^2 c^3}{qE}} \propto e^{-\pi\frac{E_c}{E}} \tag{8}$$

where we have defined the critical electric field for pair creation as $E_c = m^2c^3/q\hbar \approx 10^{18}Vm^{-1}$ if q is the (positron) elementary charge. Remarkably, we get the same qualitative result if we apply the probability for the penetration depth in a barrier $P \propto e^{-\lambda x}$, taking $x = mc/\hbar$ and the depth $\lambda = mc^2/qE$, so $\lambda x = m^2c^3/qE\hbar = E_c/E$. Reinserting a constant factor π in x we recover (8). We will use natural units $\hbar = c = 1$ in the next sections.

2.2 Formal calculation: approach 1(fermions)

In QFT we stuy the vacuum persistence amplitude

$$\langle 0|0\rangle_J = \langle 0|0\rangle^A = e^{iS_{eff}} \tag{9}$$

where we used units with $\hbar = 1$, as it is conventional in QFT. The pair production amplitude is

$$P \simeq 1 - e^{-2\Im(S_{eff})} \approx 2\Im(S_{eff}) \tag{10}$$

where formally we define

$$S_{eff} = \log\det\left[i\gamma^\mu\left(\partial_\mu - ieA_\mu\right) + m\right] \tag{11}$$

The problem is shifted to the computation of the determinant and its logarithm, above, and thus later to calculate the imaginary part of the effective action S_{eff}, i.e., $\Im(S_{eff})$. That is what Schwinger calculated indeed in his 1951 paper. We now review the main steps of the problem. First, only an additional remark about QFT: the persistence amplitude in vacuum is essentially the probability of particle creation. Mathematically,

$$\langle 0|0\rangle_J = |\langle out|in\rangle|^2 = 2\Im(S_{eff}) \approx \Im(S_{eff}) = Prob.of part.creation$$

but

$$\frac{Probability}{time\ \times\ volume} \approx \Im(\mathcal{L})$$

where $\mathcal{L}$ is the lagrangian density. In [3], Schwinger used the proper time formalism to derive the full effective action in terms of the integral

$$\Delta\mathcal{L} = -\frac{1}{8\pi^2}\int_0^\infty ds\frac{1}{s^3}e^{-m^2 s}\left[\frac{(es)^2(\mathbf{E}\cdot\mathbf{B})\Re(\cosh(esX))}{\Im(\cos esX)}\right] - 1 - \frac{2}{3}(es)^2 F \tag{12}$$

where the last two terms in the integrand function are due to renormalization, and we defined $X = 2(F + iG)$, $G = \mathbf{E}\cdot\mathbf{B}$ and $F = 1/2(B^2 - E^2)$. Now, we have two interesting limits for the above integral: i) Pure magnetic field and ii)Pure electric field. The key point comes from the latter for the common Schwinger effect, as Schwinger realized taking $B \to iE$ that

$$\Delta\mathcal{L} = -\frac{1}{8\pi^2}\int_0^\infty ds\frac{1}{s^3}e^{-m^2 s}\left(seE\cot(eEs) - 1 + \frac{1}{3}(eEs)^2\right) \tag{13}$$

We note that the integral has singularities whenever

$$s = s_n = \frac{n\pi}{eE}, \quad n = 1, 2, 3, \ldots, \infty$$

Therefore, using the residual theorem of complex variables and calculus, the integral can be evaluated considering a path above the real axis, to get an imaginary part (from the residues) equal to:

$$2\Im(\mathcal{L}) = \frac{1}{4\pi}\sum_{n=1}^{\infty} s_n^{-2} e^{-m^2 s_n} = \frac{1}{4\pi}\sum_{n=1}^{\infty}\left(\frac{eE}{n\pi}\right)^2 e^{-m^2\frac{n\pi}{eE}}$$

so then

$$2\Im(\mathcal{L}) = \frac{1}{4\pi^3}\sum_{n=1}^{\infty}(eE)^2 e^{-\frac{m^2 n\pi}{eE}}\frac{1}{n^2}$$

and finally we obtain

$$\Im(\mathcal{L}) = \frac{(eE)^2}{8\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-\frac{m^2 n\pi}{eE}} = \frac{dP[\gamma + E(field)] \rightarrow e^+e^-}{dVdT} \tag{14}$$

We can recover the heuristic (classical) result truncating the series at $n = 1$ and getting the amplitude

$$\mathcal{A} = V \cdot T\frac{(eE)^2}{8\pi^3}$$

Moreover, since $n(e^+e^-) = 2\pi\Im(\mathcal{L})$, then

$$n(e^+e^-) = V \cdot T\frac{e^2E^2}{4\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-\frac{m^2 n\pi}{eE}} = V \cdot T\frac{\alpha E^2}{4\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-\frac{m^2 n\pi}{eE}} \tag{15}$$

where the fine structure constant is $\alpha = e^2/\hbar c = e^2$, using in the last step natural units. Interesingly, the formula can be easily extended to other particles and charges with the simple rescaling of mass or charge, i.e., e.g., making $e \rightarrow Ze$, with Z the (integer) multiple of the elementary electron charge. Also, if the particle is massless (m=0), we obtain a nice formula in terms of the Riemann zeta function value:

$$n(e^+e^-) = V \cdot T\frac{\alpha E^2}{4\pi^2}\zeta(2), \quad \zeta(2) = \sum_{n=1}^{\infty}\frac{1}{n^2} \tag{16}$$

Important remark: the dependence of $\Im(\mathcal{L})$ in E is *nonperturbative*, through an exponential non-trivial function and we have included the influence of spin in the calculation through the counting factor 2 in the effective action.

2.3 Formal calculation: approach 2(bosons)

In this section, we calculate formally the Schwinger effect for bosons, although we will use the Bogoliubov transformations in order to be more precise and to prepare the path for further generalizations.

We consider a Klein-Gordon(KG) massive and spinless field in a bos of volume $V = L^3$ coupled with an external source (a force $\vec{f}(t)$). The KG field is equivalent to a Hamiltonian system formed by an infinite number of harmonic oscillators with time-dependent frequencies. The mode equations read

$$\xi_k'' + \omega_k^2 \xi = 0, \quad k \in Z \tag{17}$$

and where we define

$$\omega_k^2 = \left| \frac{2\pi \vec{k}}{L} + e\vec{f} \right|^2 + m^2$$

The equation (17) can not be solved in closed general form, so we search for solutions that satisfy the conditions $\lim_{t\to\pm\infty} \omega_k(t) = \omega_{k,\pm t}$, and they are called asymptotically free particles, also sometimes called quasiparticles. Then (17) admits solutions:

$$\xi_{in,\vec{k}}(t) = \alpha_k \xi_{out,k}(t) + \beta_k \xi^*_{out,k}(t) \tag{18}$$

with the definitions of the quantum operators

$$a_{out,k} = \alpha_k a_{in,k} + \beta_k^* a^+_{in,k}$$

Let us define $|n_k\rangle_{in} = |n_k\rangle$ to be the in-state containing n particles in the k-mode and $|n_k\rangle_{out} = |n_k)$ the out-state containing n particles in the k-mode. Then, as it is well known, cf. eg. [4],

$$|0_k\rangle = \tilde{c}_k \sum_{n=0}^{\infty} \left(\frac{\beta_k^*}{\alpha_k^*} \right) |n_k) = \tilde{c}_k \sum_{n=0}^{\infty} \left(\frac{\beta_k^*}{\alpha_k^*} \right) |n_k\rangle_{out} \tag{19}$$

$$|0_k) = c_k \sum_{n=0}^{\infty} \left(-\frac{\beta_k^*}{\alpha_k} \right) |n_k\rangle = c_k \sum_{n=0}^{\infty} \left(-\frac{\beta_k^*}{\alpha_k} \right) |n_k\rangle_{in} \tag{20}$$

where we have the relationships

$$|\tilde{c}_k|^2 = |c_k|^2 = \frac{1}{|\alpha_k|^2} \tag{21}$$

From these equations, we can see that the probability that a particle in the k-mode is produced will be given by the transition amplitude squared:

$$|(n_k\, |0_k\rangle\, |^2 = \frac{|\beta_k|^{2n}}{\left(1 + |\beta_k|^2\right)^{n+1}} \tag{22}$$

Then, the average number of particles in the k-mode would be

$$\langle 0_k | \, a^{+}_{out,k} a_{out,k} \, | 0_k \rangle = |\beta_k|^2 \tag{23}$$

Then, formally

$$\begin{aligned} P = \prod_k \left(1 + |\beta_k|^2\right)^{-1} = \exp\left[-\sum_k \log\left(1 + |\beta_k|^2\right)\right] = \\ \exp\left[-\sum_k \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} |\beta_k|^{2n}\right] \end{aligned} \tag{24}$$

where we used the expansion $\log(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$. Equation (24) gives the probability for the vacuum state to be unchanged. In fact, from equation (24) we get

$$|_{out} \langle 0|0 \rangle_{in}|^2 = \exp\left[-\sum \log(1 + N_k)\right] = \exp\left[-\sum_k \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} N_k^{n-pairs}\right] \tag{25}$$

or

$$|_{out} \langle 0|0 \rangle_{in}|^2 = \exp\left[-\frac{2TL^3E^2\alpha}{8\pi^3\hbar} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \exp\left(-\frac{n\pi m^2 c^4}{\hbar c e E}\right)\right] \tag{26}$$

and recall again that the critical field is defined as $E_c = m^2c^3/q\hbar$. The equation (26) is the Schwinger formula for bosons.

3 The meaning of Schwinger's formula

In spite of its importance and being one of the most cited papers in Physics, the new and reborn interest in the mechanism must confront some misconceptions. The main one is the interpretation of Schwinger's formula itself. It is often unknown for non-experts in the field. As several authors have pointed out and remarked, Schwinger's interpretation is flawed (see, e.g., [5, 6, 7, 8, 9]. Here, we will follow [5]. Thus, for the vacuum probability

$$P_{vac}(t) = |\langle vac| \, U(t) \, |vac\rangle|^2 = \exp(-\omega V t) \tag{27}$$

with

$$\omega = \frac{q^2E^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left[-\frac{n\pi m^2}{qE}\right] \tag{28}$$

Using the polylogarithm function

$$Li_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s}$$

we also get the nice expression

$$\omega = \frac{(qE)^2}{4\pi^3} Li_2\left(\exp\left(-\frac{\pi m^2}{qE}\right)\right) \tag{29}$$

Now, we can discretize the probability in the following way

$$P_{vac} = e^{-\int d^4x \omega(x)} = \lim_{\Delta v \to 0} \prod_{i=1}^{\infty} e^{-\omega(x_i)\Delta v_i} = \prod_{i=1}^{\infty} (1 - \omega(x_i)\Delta v_i) \tag{30}$$

Now, we can understand why ω *is not* the rate of pairs created since (9),(10) gave the right answer for the rate (in the case of d=3+1 QED)

$$\Gamma = \frac{(qE)^2}{4\pi^3} \exp\left[-\frac{\pi m^2}{qE}\right] = \frac{\langle N \rangle}{VT} \neq \omega \tag{31}$$

Indeed, the proper and accurate interpretation of Schwinger's formula is that it provides the *relative* probability of pair production, and the rate Γ can be obtained as the first term in the series for ω. Higher order terms in the expansion give the rate of multiple pairs creation. Cohen showed this too [5] in a simple toy model: d=1+1 QED. He obtained:

$$\omega_{ferm}^{1+1QED} = \frac{qE}{2\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left[-\frac{n\pi m^2}{qE}\right] = -\frac{qE}{2\pi} \log\left(1 - e^{-\frac{\pi m^2}{qE}}\right) \tag{32}$$

Hence, we see that the rate is

$$\Gamma^{1+1QED} = \frac{\langle N \rangle}{LT} = \frac{qE}{2\pi} \exp\left[-\frac{\pi m^2}{qE}\right] \neq \omega^{1+1QED} \tag{33}$$

In summary, Schwinger's formula gives the relative probability to vacuum that a pair be created per unit of volume and time (that is given by ω) and the average number of pairs produced per unit of volume and time(that is given by the rate Γ). This two concepts are not the same in general and they can be confused if we are careless. See e.g. [7, 8, 9] for a rigurous proof.

4 Generalizations

The Schwinger's mechanism is a completely general non-perturbative effect of gauge theories. It can be calculated for both scalar and fermion fields in QED in any dimension of spacetime [10, 11]. In this section, we first review the main formulae of [10] and then we will show the general formulae due to its great interest and beautiful mathematical expression. Note that the field is generally not homogeneous in $\mathcal{L}$ but it is studied the case where E is constant as a particular case.

4.1 Spinor QED

If d=2+1, we have following [10]

$$\mathcal{L}_f^{2+1QED}(E) = \frac{q^2 e^{-i\pi/4}\left(\partial_{\|}E\right)^2}{4\left(4\pi|qE|\right)^{3/2}} \int_0^\infty d\omega \frac{1}{\sqrt{\omega}} e^{-\frac{im^2\omega}{|qE|}} \frac{d^3}{d\omega^3}\left(\omega\coth\omega\right) \tag{34}$$

Mimicking the Schwinger's strategy, we use residues to evaluate the above integral to extract the imaginary part of the effective lagrangian for constant electric fields. In this case,

$$\Im(\mathcal{L}_f^{2+1QED}(E)) = \frac{|qE|^{3/2}}{4\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \exp\left(-\frac{\pi m^2 n}{qE}\right) \tag{35}$$

$$\Im(\mathcal{L}_f^{2+1QED}(E)) = \frac{|qE|^{3/2}}{4\pi^2} Li_{3/2}\left(\exp\left[-\frac{\pi m^2}{|qE|}\right]\right) \tag{36}$$

and where we have defined $\partial_{\|}E = \partial_0 E\partial_0 E - \partial_1 E\partial_1 E$ and again we finally wrote the result in terms of the polylogarithm $Li_s(z)$. If now we write the resulting expression for fourdimensional $d = 3+1$ QED, we get similarly the previous result by Schwinger:

$$\Im(\mathcal{L}_f^{3+1QED}(E)) = -\frac{iq^2\left(\partial_{\|}E\right)^2}{(8\pi)^2|qE|} \int_0^\infty \frac{d\omega}{\omega} e^{-\frac{im^2\omega}{|qE|}} \frac{d^3}{d\omega^3}(\omega\coth\omega) \tag{37}$$

and for constant electric field, we obtain the result

$$\Im(\mathcal{L}_f^{3+1QED}(E)) = \frac{(qE)^2}{8\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\frac{m^2\pi n}{|qE|}} \tag{38}$$

$$\Im(\mathcal{L}_f^{3+1QED}(E)) = \frac{(qE)^2}{8\pi^3} Li_2\left(\exp\left[-\frac{\pi m^2}{|qE|}\right]\right) \tag{39}$$

4.2 Bosonic QED

In d=2+1 dimensions of spacetime, we obtain the following effective lagrangian

$$\Im(\mathcal{L}^{QED}_{b,2+1d}(E)) = -\frac{e^{-i\pi/4}q^2\left(\partial_{\parallel}E\right)^2}{16\pi|qE|^{3/2}}\int_0^{\infty}\frac{d\omega}{\omega}e^{-\frac{im^2\omega}{|qE|}}\left(\frac{d^3}{d\omega^3}+\frac{d}{d\omega}\right)\frac{\omega}{\sinh\omega} \quad (40)$$

and for constand field this turns to be

$$\Im(\mathcal{L}^{QED}_{b,2+1d}(E)) = \frac{|qE|^{3/2}}{8\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{3/2}}\exp\left[-\frac{\pi m^2 n}{|qE|}\right] \quad (41)$$

$$\Im(\mathcal{L}^{QED}_{b,2+1d}(E)) = -\frac{|qE|^{3/2}}{8\pi^2}Li_{3/2}\left(-\exp\left[-\frac{\pi m^2}{|qE|}\right]\right) \quad (42)$$

By the other hand, if we are in a (3+1)d spacetime, the equivalent expressions are

$$\Im(\mathcal{L}^{QED}_{b,3+1d}(E)) = \frac{iq^2\left(\partial_{\parallel}E\right)^2}{2\left(8\pi\right)^2|qE|}\int_0^{\infty}\frac{d\omega}{\omega}e^{-\frac{im^2\omega}{|qE|}}\left(\frac{d^3}{d\omega^3}+\frac{d}{d\omega}\right)\frac{\omega}{\sinh\omega} \quad (43)$$

and

$$\Im(\mathcal{L}^{QED}_{b,3+1d}(E)) = \frac{(qE)^2}{16\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\exp\left[-\frac{\pi m^2 n}{|qE|}\right] \quad (44)$$

$$\Im(\mathcal{L}^{QED}_{b,3+1d}(E)) = -\frac{(qE)^2}{16\pi^3}Li_2\left(-\exp\left[-\frac{\pi m^2}{|qE|}\right]\right) \quad (45)$$

4.3 Schwinger effect in higher dimensional QED

The previous results can be generalized to higher dimensional QED. The main formulae were given in previous works, e.g. in [11]. There, the authors related the single and multiple pair production of bosons and fermions with the single and multiple instantons in higher dimensional non-perturbative QED at finite temperature. In (d+1) Minkovski spacetime ($D = d+1$) we obtain for bosons (where the spin is denoted by s), with $\omega = 2\Im(\mathcal{L}_{eff})$ as before, and the volume splitting $V = V_{\parallel}V_{\perp}$, such as $\int d\omega = V_{\parallel}qE$. Therefore, for bosons in higher dimensions, we deduce that

$$\omega_{b,d+1} = \frac{(2s+1)V_{\perp}}{V}\int\frac{d\omega}{(2\pi)^d}\int d\vec{k}_{\perp}^{d-1}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}e^{-\frac{\vec{k}_{\perp}^2\pi n}{qE}}e^{-\frac{m^2\pi n}{qE}} \quad (46)$$

and after making the integration we get

$$\omega_{b,d+1}(E) = \frac{(2s+1)}{(2\pi)^d}\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{qE}{n}\right)^{\frac{d+1}{2}}\left(\exp\left[-\frac{\pi m^2 n}{qE}\right]\right) \tag{47}$$

$$\omega_{b,d+1}(E) = -\frac{(2s+1)\,|qE|^{\frac{d+1}{2}}}{(2\pi)^d} Li_{\frac{d+1}{2}}\left(-\exp\left[-\frac{\pi m^2}{|qE|}\right]\right) \tag{48}$$

For fermions in higher dimensional QED, using a $d+1$ Minkovski spacetime, we algo get

$$\omega_{f,d+1} = \frac{(2s+1)V_\perp}{V}\int\frac{d\omega}{(2\pi)^d}\int d\vec{k}_\perp^{d-1}\sum_{n=1}^{\infty}\frac{1}{n}e^{-\frac{\vec{k}_\perp^2\pi n}{qE}}e^{-\frac{m^2\pi n}{qE}} \tag{49}$$

and

$$\omega_{f,d+1}(E) = \frac{(2s+1)}{(2\pi)^d}\sum_{n=1}^{\infty}\left(\frac{qE}{n}\right)^{\frac{d+1}{2}}\left(\exp\left[-\frac{\pi m^2 n}{qE}\right]\right) \tag{50}$$

$$\omega_{f,d+1}(E) = \frac{(2s+1)\,|qE|^{\frac{d+1}{2}}}{(2\pi)^d} Li_{\frac{d+1}{2}}\left(\exp\left[-\frac{\pi m^2}{|qE|}\right]\right) \tag{51}$$

The mathematical beauty and elegance of equations is astonishing, specially (48) and (51). Other generalizations, in particular with simultaneous electric and magnetic strong fields $(\vec{E},\vec{B})$ do exist (see, e.g., [12]), but we let this fascinating topic with the shocking polylogarithmic expressions above. Only a final important remark: for strong electric fields E, ω^d exceed the given production rate Γ^d by a factor of the order $\zeta(\frac{d+1}{2})$, as large as $\zeta(2)\sim 1.64$ in d=3, D=4=3+1.

5 Experimental searches

In the nice talk [13], it was explained the main problems related with the observation of Schwinger's effect:

- Available experimental electric fields $\vec{E}$ are too small compared with respect to the critical field $E_c = \dfrac{m^2c^3}{qe\hbar}$.
- Different experimental set-ups involving strong electric fields in laser pulses ($\vec{E} = \vec{E}(t)$), at last, depend on complicated dynamics.

- Electron mass put a strong bound on the critical field since it is very small (the key point to understand why Schwinger effect is hard to observe). Then the exponential suppression of both ω and Γ makes the physical observation very challenging.

However, some interesting and forthcoming projects with laser facilities like ELI (Extreme Light Infrastructure) will try to manage its realization. In the same talk, previously mentioned, Cohen et al. proposed to use an analogue system in condensed matter: the well-known graphene. The reason for this proposal are the following:

- Charged massless (or almost massless) fermions imply the exponential approaches to the unit and the effect would be easier to observe due to the exponential absence.

- Graphene systems, dut to its importance and present activity, provide a fermioni model of quasiparticles in low dimensional, e.g. 2+1d, systems whose spectrum is very similar to the relativistic massless (or light) charged fermions. In fact, in a potential with hexagonla symmetry (a honneycomb lattice) we obtain the spectrum:

$$\varepsilon^2 = \tilde{c}^2 \left(\Delta p_x^2 + \Delta p_y^2\right) + \dots \tag{52}$$

  so heuristically adjusting the dimensional factors, we could get the Schwinger rate:

$$\Gamma^{2+1}_{graphene} = f\frac{(qE)^{3/2}}{4\pi^2}\zeta\left(\frac{3}{2}\right) = f\frac{(qE)^{3/2}}{4\pi^2}\sum_{n=1}^{\infty}\frac{e^{-\frac{m\pi^2 n}{qE}}}{n^{3/2}} \tag{53}$$

  Therefore, if we insert dimensions, for $m = 0$, $f = 4 = 2(2(1/2) + 1)$, we easily obtain

$$\Gamma^{2+1}_{graphene} = \frac{(qE)^{3/2}}{\pi^2\hbar\tilde{c}^{1/2}}\zeta\left(\frac{3}{2}\right) \tag{54}$$

  where $\zeta(\frac{3}{2}) \approx 2.612$.

6 Summary and conclusions

We have studied and reviewed the Schwinger effect for bosons and fermions from different viewpoints. Our calculations were done by a heuristical path and a formal approach. We revised the Schwinger's original paper and understood

$$2\Im\left(\mathcal{L}_{eff}\right) = \begin{cases} -tr_p(1-n_p), \text{for fermions} \\ +tr_p(1+n_p), \text{for bosons} \\ tr_p(n_p), \text{for Maxwell-Boltzmann particles} \end{cases}$$

as a series that gives us the number of pairs created (as long as $n_p << 1$) and it is independent of the statistics. Higher terms can be understood as a coherent production of multiple pairs in a given spacetime volumen for any dimension. The origin of this effect is a non-perturbative phenomenon, and hence, it can not be reproduced by any perturbative analysis. Indeed, the Euler-Heisenberg lagrangian and its generalizations is in general a divergent series from which we can make a Borel sum so that if

$$f(-g) = \sum_{n=0}^{\infty} n! g^n = \frac{1}{g}\int_0^{\infty} dt \frac{e^{-t/g}}{1-t} \tag{55}$$

the imaginary part of $f(-g)$, the Borel sum, has a pole with residue

$$Res(f) = -\frac{1}{g}e^{-1/g} \tag{56}$$

and then

$$\Im(f(-g)) = \frac{\pi}{g}e^{-1/g} \tag{57}$$

is a non analytical function of g. This effect does not appear in perturbation theory. We have also used some topics of QFT in curved spacetime in our formal approach and we have written the final expressions for the Schwinger effect in any dimension with the aid of polylogarithms.

The experimental searches for this important mechanism are being pursued yet and we have presented some novel and recent remarks concerning the measurment of the masses of the pairs and the experimental searches in low dimensional systemas such as graphene. It is interesting that there could be other unknown ways to test the effect. One can imagine that in the Universe can be astrophysical objects or phenomena that could be able to produce the large electric fields that are required to produce this non-trivial SM effect. But, as far as we know, that search has not been done. Likely, more interplay between theory (likely more condensed matter physics and astrophysics) and experimental sides will be necessary in order to confirm the non-perturbative predictions of QED.

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