Extended Relativity: Beyond

(Based on previous works by *C. Castro* and *M. Pavšič*)
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CONTENTS

EDGES OF PHYSICS (I): GR AND SM

⋆ 20th/currrent century physics coded into 2 big EFT (up to $E \sim 100GeV - 1TeV$ and about $10^{-18}m < \lambda < R_{Obs}$) :

GR/Standard Gravity (SG)

$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ with $\mathcal{L}_{GR} = \mathcal{L}_{EH} + \mathcal{L}_M$

QFT gauge field theory/Standard Model (SM)

$$\boxed{L_{SM} = L_\psi + L_{gauge} + L_Y + L_{Higgs}}$$

$$L_\psi = i\overline{\Psi}\not{D}\Psi + h.c.$$

$$L_{gauge} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$L_Y = Y_{ij}\phi\overline{\Psi}_i\Psi_j + h.c.$$

$$L_{Higgs} = |D_\mu\phi|^2 - V(\phi)$$

EDGES OF PHYSICS (II): ISSUES/QUESTIONS

⋆ This can NOT be the end of the story. A list:

1. Dark Matter and Dark Energy. What are they? Particles and/or Modified Gravity?
2. The Cosmological Constant (dark energy?) problem.
3. Origin of mass (how to explain why the Yukawa couplings and masses are those we observe?).
4. Black Hole physics: where does BH entropy come from? Information paradox+spacetime singularities…
5. Quantum Mechanics and its foundations. Is it geometry? Is it (really) fundamental or emergent?
6. Is spacetime itself fundamental or emergent?
7. Gravitational, strong and electroweak forces not unified.

EDGES OF PHYSICS (III): BEYOND GR/SM

⋆ Go beyond. QG and U. Main approaches BGR and BSM:

1. Superstrings/M-theory.
2. Loop Quantum Gravity.
3. Other approaches, ideas, and frameworks.
   - CFT, GUT's, NC geometry, twistors.
   - Phenomenology of QG.
   - Higher Spin Theories, Generalized UP, analogue models, SME.
   - World crystals, holography, gauge/gravity correspondence, emergent spacetime, QIT "spacetime from entanglement", ...
4. Deformations and extensions of SR, GR and QM. There are nontrivial extensions of relativity and other relativities (doubly/triply special relativity)!

This talk: The Extended Relativity in C-spaces "state of art" and its own "beyond"

ER (I): Why?

Six good reasons to study Extended Relativity and ER in C-spaces:

- Not too many people out there doing it! No competitors! (This is a good one!)
- New ways to enlarge relativity/gauge symmetries. String theory/M-theory and other main approaches still lack a unifying principle (cf. equivalence principle, Lorentz invariance, diff. invariance,…)
- Derive relationships with other known major/minor approaches
- Alternative tool to compactification/extra D/unification
- Clifford algebras seem to be important too in QIT
- Create new predictions to be tested in experiments and explore new paths towards unification (Not bad!)

ER(II): POLYVECTORS AND ER RISE

Castro 98/99, Pezzaglia 99, C. Castro/M. Pavšič 2005: Extended relativity theory in C-spaces generalizes of the notion of the interval in Minkowski space to a manifold we call Clifford space (C-space) and naturally requires extended objects. *Matej Pavšič (IARD 2002)*: polydimensional relativity+C-space as the "arena" of physics.

What is a polyvector?

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What is a polyvector?

The Clifford valued polyvector $X = X^M E_M$ is defined as:

$$X = \sigma \underline{1} + x^{\mu}\gamma_{\mu} + x^{\mu\nu}\gamma_{\mu}\wedge\gamma_{\nu} + \ldots + x^{\mu_1\mu_2\ldots.\mu_D}\gamma_{\mu_1}\wedge\gamma_{\mu_2}\ldots.\wedge\gamma_{\mu_D}$$

Interpretation: a "point" in C-space has coordinates X^M and basis E_M. The series ends at a *finite* grade depending on the dimension D. A Clifford algebra $Cl(r,q)$ with $r+q=D$ has 2^D basis elements. Clifford algebra/geometric calculus use the product $ab = a\cdot b + a\wedge b$.

FROM MINKOWSKI TO CLIFFORD SPACETIME (I)

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$$\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}$$

- Einstein introduced the speed of light as a universal *absolute* invariant in order **to unite** space with time (to match units) in the Minkwoski space interval:

$$ds^2=c^2dt^2+dx_idx^i$$

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- The C-space interval

The C-space interval generalizes Minkovskian spacetime:

$$dX^2 = d\sigma^2 + dx_\mu dx^\mu + dx_{\mu\nu}dx^{\mu\nu} + \ldots$$

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2. The symbol $X^{\dagger}$ denotes the *reversion* operation and involves reversing the order of all the basis γ^{μ} elements in the expansion of X.It is the analog of the transpose (Hermitian) conjugation
3. The C-space metric associated with a polyparticle motion is then :

$$|dX|^2 = G_{MN}\, dX^M dX^N \quad (1)$$

where $G_{MN} = E^{\dagger}_M * E_N$ is the C-space metric.

$$|dX|^2 = d\sigma^2 + L^{-2} dx_{\mu} dx^{\mu} + L^{-4} dx_{\mu\nu} dx^{\mu\nu} + \ldots + L^{-2D} dx_{\mu_1 \ldots \mu_D}\, dx^{\mu_1 \ldots \mu_D} \quad (2)$$

From Minkowski to Clifford spacetime (III)

- *Neccesary* introduction: Planck scale L. It is **length** parameter is needed in order to tie objects of different dimensionality together: 0-loops, 1-loops,..., p-loops.
- This procedure can be carried to all closed p-branes (p-loops) where the values of p are $p = 0, 1, 2, 3, \ldots$. The $p = 0$ value represents the center of mass and the coordinates $x^{\mu\nu}, x^{\mu\nu\rho}\ldots$

MOTION IN C-SPACE(I)

Line element and polymomentum

$$dX^A dX_A = d\sigma^2 + \left(\frac{dx^0}{L}\right)^2 - \left(\frac{dx^1}{L}\right)^2 - \left(\frac{dx^{01}}{L^2}\right)^2 \cdots + \left(\frac{dx^{12}}{L^2}\right)^2 -$$

$$\left(\frac{dx^{123}}{L^3}\right)^2 - \left(\frac{dx^{0123}}{L^4}\right)^2 + \ldots = 0$$

- Vanishing of $\dot{X}^B \dot{X}_B$ is equivalent to vanishing of the above C-space line element and by "..." we mean the terms with the remaining components such as x^2, x^{01}, x^{23},..., x^{012}, etc.
- The C-space metric is $G_{MN} = E_M^\dagger * E_N$ and if the dimension of spacetime is 4, then x^{0123} is the highest grade coordinate.

Motion in C-space(II)

Polyvelocity

$$V^2 = -\left(L\frac{d\sigma}{dt}\right)^2 + \left(\frac{dx^1}{dt}\right)^2 + \left(\frac{dx^{01}}{L^2}\right)^2 \ldots$$
$$-\left(\frac{1}{L}\frac{dx^{12}}{dt}\right)^2 + \left(\frac{1}{L^2}\frac{dx^{123}}{dt}\right)^2 + \left(\frac{1}{L^3}\frac{dx^{0123}}{dt}\right)^2 - \ldots$$

We find that

- The maximum speed $V^2 = c^2$ in C-space depends on extra r-vector quantities.
- The maximum speed squared V^2 contains components of the 1-vector velocity dx^1/dt, but also the multivector $dx^{12}/dt, dx^{123}/dt, \ldots$ The following special cases in C-space are different from zero, are of particular interest:

MOTION IN C-SPACE(III)

Maximum 1-vector speed

$$\frac{dx^1}{dt} = c = 3.0 \times 10^8 m/s$$

Maximum 3-vector speed

$$\frac{dx^{123}}{dt} = L^2 c = 7.7 \times 10^{-62} m^3/s$$

And we have as well...

Motion in C-space(IV)

Maximum 3-vector *diameter* speed and Maximum 4-vector speed

$$\frac{d\sqrt[3]{x^{123}}}{dt} = 4.3 \times 10^{-21} m/s \quad \frac{dx^{0123}}{dt} = L^3 c = 1.2 \times 10^{-96} m^4/s$$

Remark: in addition to this, you can also get maximal limits to n order derivatives given by

$$Max\left(\frac{d^{n+1}x}{dt^{n+1}}\right) \leq c\left(\frac{c}{L}\right)^n$$

Remark (II): hint of a high derivative extension of relativity? Have you ever heard about tachyons and epitachyons?

ER: MAXWELL ELECTRODYNAMICS(I)

C-space gauge field theory electromagnetism

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3. Defining the C-space operator $(M, N = 1, 2, \ldots, 2^D)$

$$d = E^M \partial_M = \underline{1} \partial_\sigma + \gamma^\mu \partial_{x_\mu} + \gamma^\mu \wedge \gamma^\nu \partial_{x_{\mu\nu}} + \ldots$$

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4. The generalized field strength in C-space is:

$$F = dA = E^M \partial_M (E^N A_N) = E^M E^N \partial_M A_N =$$
$$\frac{1}{2}\left\{E^M, E^N\right\}\partial_M A_N + \frac{1}{2}\left[E^M, E^N\right]\partial_M A_N =$$
$$\frac{1}{2}F_{(MN)}\left\{E^M, E^N\right\} + \frac{1}{2}F_{[MN]}\left[E^M, E^N\right]$$

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$$\int A_M dX^M = \int [DX] J_M A^M$$

C-space Maxwell Electrodynamics(IV): equations and generalizations

C-space Maxwell equations

$$\partial_M F^{[MN]} = J^N \quad \partial_N \partial_M F^{[MN]} = 0 = \partial_N J^N = 0$$

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$$F = DA = (dA + A \bullet A) \quad E_M \bullet E_N = E_M E_N - (-1)^{s_M s_N} E_N E_M$$

GENERALIZED POLYVECTOR VALUED GAUGE FIELDS

$$\mathbf{X} = \varphi\,\mathbf{1} + x_\mu\,\gamma^\mu + x_{\mu_1\mu_2}\,\gamma^{\mu_1}\wedge\gamma^{\mu_2} + x_{\mu_1\mu_2\mu_3}\,\gamma^{\mu_1}\wedge\gamma^{\mu_2}\wedge\gamma^{\mu_3} + \ldots$$

E.g.: Polyvector valued gauge field in Cl (5,C)

$\mathcal{A}_M(\mathbf{X}) = A^I_M(\mathbf{X})\,\Gamma_I$ is spanned by $16+16$ generators. The expansion of the poly-vector $\mathcal{A}^I_M$ is also of the form

$$\mathcal{A}^I_M = \Phi^I\,\mathbf{1} + A^I_\mu\,\gamma^\mu + A^I_{\mu_1\mu_2}\,\gamma^{\mu_1}\wedge\gamma^{\mu_2} + A^I_{\mu_1\mu_2\mu_3}\,\gamma^{\mu_1}\wedge\gamma^{\mu_2}\wedge\gamma^{\mu_3} + \ldots$$

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C-SPACE KLEIN-GORDON AND DIRAC WAVE EQUATIONS

Polymomentum correspondence principle

$$P_A \to -i\frac{\partial}{\partial X^A} = -i\left(\frac{\partial}{\partial \sigma}, \frac{\partial}{\partial x^\mu}, \frac{\partial}{\partial x^{\mu\nu}}, \ldots\right) \quad \Psi(x^\mu) \to \Psi(x^A)$$

C-space Klein-Gordon wave equation

$$\left(\frac{\partial^2}{\partial \sigma^2} + \frac{\partial^2}{\partial x^\mu \partial x_\mu} + \frac{\partial^2}{\partial x^{\mu\nu} \partial x_{\mu\nu}} + \ldots + M^2\right)\Phi = 0$$

C-SPACE KLEIN-GORDON AND DIRAC WAVE EQUATION (II)

C-space Dirac wave equation

$$-i\left(\frac{\partial}{\partial\sigma}+\gamma^\mu\frac{\partial}{\partial x_\mu}+\gamma^\mu\wedge\gamma^\nu\frac{\partial}{\partial x_{\mu\nu}}+\ldots\right)\Psi=M\Psi$$

Note we used natural units in which $\hbar=1, c=1$

1ST LINK: MAXIMAL ACCELERATION/TENSION/FORCE/POWER

- Caianiello's QM geometry as phase-spacetime includes

$$a_C = 2\frac{mc^3}{\hbar} = 2\frac{Ec}{\hbar}$$

- Born's reciprocal relativity.
- Maximal Force implies a maximal power (e.g. Schiller 2006). The recent LIGO detection of GW is only about 10^{-2} the maximal power. **Hope:** we aspire to test maximal power (force?) with GW radiation in the future!
- Maximal acceleration, via EP, implies a maximal, critical, strong gravitational field. The SM in strong fields (Schwinger effect) remains as challenge!

2ND LINK: EMERGENT SPACETIME AND COMPLEXITY

- Role of "emergence": emergent spacetime from entanglement? Is quantum entanglement the key?
- Complexity and gravity interplays. Indeed, Susskind et al. recently related complexity with action and they got the rate:

$$\frac{d\text{Complexity}}{dt} \leq \frac{2M}{\pi\hbar}$$

  It suggests a link with maximal acceleration as well after rescaling with π, c^3!

3RD LINK: OTHER RELATIVITIES (OR)

Past works on (forgotten) OR. A simple (non-exhaustive) list includes (choose one or many!):

- Born seminal work on reciprocal relativity.
- Fantappie's final relativity and Arcidiacono's projective relativity (dS like).
- Kalitzin's multitemporal relativity (1975 book).
- Barashenkov's 6D relavitivy.
- Cole's 6D spacetime relativity and cellular spacetime.
- Bogoslovski's anisotropic relativity (Very Special Relativity).
- De Sitter relativity (doubly/triply SR).
- C. Nassif's minimal velocity relativity.
- Gogberashvili's octonionic relativity.
- ...Are all wrong or some of their ideas are right indeed?

BEYOND ER: HINTS OF A NEW ER

Everything so far sounds good, what is the problem with ER? A critical view:

- No clear principle(s) but points into it(them) in a sense: why is fundamental scale $L=L_p$? What about a dual extension with MAXIMAL/dS length $L=L_\Lambda$?
- Transitions between different signatures not understood yet.
- The Clifford group choice: we can not choose a reason of why to pick one instead any other.
- Similar issue to theories of strings/branes: no hints of the emergence of multiple energy or mass scales.

BUT…ER gives hints and extra suggestions!

HINTS OF A NEW ER (II)

Classical Mechanics is based on the Poincare-Cartan two-form

$$\omega_2 = dx \wedge dp$$

There $p = \dot{x}$. Quantum mechanics is secretly a subtle modification of this. By the other hand, the so-called Born-reciprocal relativity is based on the "phase-space"-like metric

$$ds^2 = dx^2 - c^2 dt^2 + Adp^2 - BdE^2$$

and its full space-time+phase-space extension:

$$ds^2 = dX^2 + dP^2 = dx^\mu dx_\mu + \frac{1}{\lambda^2} dp^\nu dp_\nu$$

HINTS OF A NEW ER (III)

Extension of Born's reciprocal relativity in C-spaces based on higher accelerations IS an interesting open problem. E.g.: take $ds^2 = dx^2 + dp^2 + df^2$. We have an invariant and likely hidden Nambu dynamics

$$\omega_3 = dX \wedge dP \wedge dF$$

Question: What is the symmetry group or invariance of the above (n+1)-form and whose intersection with the $SO(D(n+1))$ group gives the higher order metaplectic group?

$$\omega_{n+1} = dx \wedge dp \wedge d\dot{p} \wedge \cdots \wedge dp^{(n-1)}$$

where we include up to $(n-1)$ derivatives or equivalently

$$\omega_{n+1} = dx \wedge d\dot{x} \wedge d\ddot{x} \wedge \cdots \wedge dx^{(n)}$$

A unified framework for higher derivative theories?

HINTS OF A NEW ER (IV)

"New" relativities and some extensions of relativity do exist and they include several ingredients and hypothesis to be tested. Furthermore, I also propose:

Ultimate Relativity (UR) conjecture

There is a extended relativity with min/max values of any n-th derivative of coordinates (also for polyvector derivatives in C-spaces).

UR: german word, "original". Also, in Geology, "the first (prime) supercontinent".
Remark: This is not really "completely new" but a reboot and revival of an older idea, cf. "final relativity" (Fantappie, Arcidiacono) and more recently Wilczek's "total relativity".

DM AS MINIMAL ACCELERATION DYNAMICS?

Suppose there is a minimal acceleration a_0 (minimal force F_0). Then:

$$\frac{v^2}{R} = G\frac{M}{R^2} + a_0$$

and from this, by simple squaring, you obtain

$$v^4 = G^2M^2R^{-2} + a_0^2R^2 + 2GMa_0$$

In the limit $G^2, a_0^2 << 1$, you get the phenomenological law

$$\boxed{v^4 = 2GMa_0}$$

Idea: DM, even if real, could be hinting a minimal acceleration (MOND-like) dynamics.

DM+DE AS MINIMAL ACCELERATION DYNAMICS + MAXIMAL LENGTH?

Suppose (with $c = 1$) there is a minimal acceleration a_0 (minimal force F_0) and a cosmological constant (de Sitter radius):

$$\frac{v^2}{R} = G\frac{M}{R^2} + a_0 + \Lambda R$$

and from this, by simple squaring, you obtain

$$v^4 = G^2M^2R^{-2} + a_0^2R^2 + 2GMa_0 + 2GMR\Lambda + 2a_0\Lambda R^3 + \Lambda^2R^4$$

In the limit $G^2, a_0^2, \Lambda << 1$, you get the phenomenological law

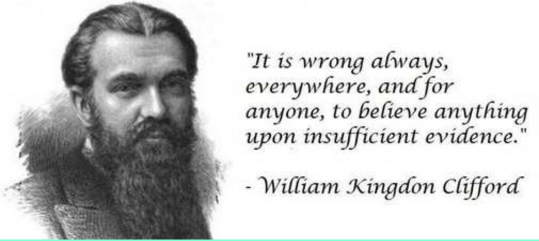
$$\boxed{v^4 = 2GMa_0}$$

Idea: DM, DE, even if real, could be hinting a (MOND-like) minimal acceleration +Maximal length dynamics. Does MOND fail (when it does) because we ignore extra terms?

SUMMARY AND OVERVIEW

1. There are multiple advantages of recurring to C-spaces. Not covered here: gravity with torsion, YM fields and nonabelian EM with CS terms,. . .
2. Every physical quantity is a polyvector! Polydimensional and signature relativity.
3. C-space dynamics (motion and electrodynamics) is richer than ordinary Minkovskian dynamics.
4. Field equations (KG, Dirac,. . .) in C-space.
5. A maximal force (accelaration) principle and phase space duality are present in the theory.
6. Is Max. acceleration related to max. complexity?
7. Born's reciprocal relativity + Nambu dynamics and likely Finsler-like higher order geometries (sometimes referred as Kawaguchi geometry) seems to be relevant there.
8. A higher order M(inimal)-maximal n-order high derivative theory? *UR* conjecture.

CLOSING AND ACKNOWLEDGEMENTS



I am grateful for your attendance

THANK YOU FOR YOUR ATTENTION! (*Hvala!*)

HINTS OF (BEYOND) ER IN FUTURE PAST DAYS(I)

ER and beyond ER are ideas something anticipated and have visited us cyclically.
A notable recent example:

John Preskill
18 April at 00:06 · Twitter ·

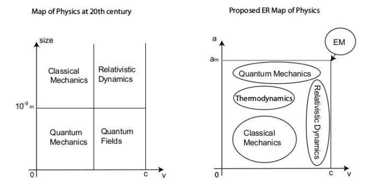
Vafa: "What is an invariant concept in string theory? The answer is, I don't know." Many hints for a new principle of relativity. #apsapril

One from IARD, not long, long ago:

IARD2012 IOP Publishing
Journal of Physics: Conference Series **437** (2013) 012017 doi:10.1088/1742-6596/437/1/012017

Extending the Relativity of Time

Yaakov Friedman



ABOUT CAIANIELLO AND THE MAXIMAL ACCELERATION PRINCIPLE

Maximal Proper Acceleration Relative to the Vacuum.

H. E. BRANDT

U.S. Army Electronics Research and Development Command
Harry Diamond Laboratories - Adelphi, MD 20783

(ricevuto il 2 Giugno 1983)

PACS. 04.60. – Quantum theory of gravitation.

Reciprocally, the equivalence principle then suggests that the physical basis for a maximal proper acceleration relative to the vacuum may be the associated breakdown of the very concept of acceleration due to the drastic topological alteration of the classical space-time structure occurring at such an extreme acceleration.

SAKHAROV understood that the arguments leading to the maximum temperature T_{max} of thermal radiation may be inconsistent, if the laws of physics require modification at the associated high density of one quantum per Planck volume (4). At such densities there may be no stationary states, and then the equilibrium thermodynamic concept of temperature as the partial derivative of energy with respect to entropy at constant volume ceases to apply (10). These same reservations must also be placed on a possible maximum proper acceleration a_0 relative to the vacuum. However, if the above reasoning is upheld, then the concept of such a limiting acceleration may lead to new space-time structures and relationships even more revolutionary than those associated with a limiting velocity.

The cool and important remark:

is upheld, then the concept of such a limiting acceleration may lead to new space-time structures and relationships even more revolutionary than those associated with a limiting velocity.

Beyond ER: a higher order M(in-Max)-theory of ER?

Question: What about a generalized relativistic dynamics for $E = \Gamma mc^2$, using "duality" and "symmetry", such any derivative appears on equal footing? Say

$$\Gamma(X^2, V^2, A^2, \ldots) = \frac{\sqrt{1-\frac{l_0^2}{X^2}}\sqrt{1-\frac{c_0^2}{V^2}}\sqrt{1-\frac{a_0^2}{A^2}}\cdots}{\sqrt{1-\frac{X^2}{L_\Lambda^2}}\sqrt{1-\frac{V^2}{C^2}}\sqrt{1-\frac{A^2}{A_m^2}}\cdots}$$

Can we test it? Is it crazy enough to be true or useful for high derivative theories? *Note:* Caianiello's epitachyons are entities with $A > A_m$.

References and related work

This work is based on the review ***" THE EXTENDED RELATIVITY THEORY IN CLIFFORD SPACES"*** by C. Castro, M. Pavšič. *Progress in Physics* **1** (2005) 31 , and the paper **"The Clifford Space Geometry of Conformal Gravity and** $U(4) \times U(4)$ **Yang-Mills Unification"** by Carlos Castro, *International Journal of Modern Physics of Modern Physics A*.

Further bibliography: M.Pavšič, *The Landscape of Theoretical Physics: A Global View, From Point Particles to the Brane World and Beyond, in Search of a Unifying Principle* (Kluwer 2001); W.Pezzaglia [arXiv: gr-qc/9912025];I. R. Porteous, *Clifford algebras and Classical Groups* (CUP, 1995); S. Low: Jour. Phys **A** Math. Gen **35**, 5711 (2002); JMP. **38**, 2197 (1997); J. Phys. **A 40** (2007) 12095; arXiv.org : 0806.4794; C. Castro, Phys Letts **B 668** (2008) 442,...I also recommend *Notes on several phenomenological laws of quantum gravity* by Jean-Philippe Bruneton, ArXiv eprint, https://arxiv.org/abs/1308.4044