Gravitational Waves: yummy pieces

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Abstract

We provide an elementary introduction to black holes, gravitational waves and what lies behind them.

1 Introduction and short outline

Gravitational waves are ripples in space-time, perturbations of a constant diagonal metric, i.e.:

$$g = g_0 + h(x,t), \quad h(x,t) << 1 \tag{1}$$

Outline of this short review:

- Black holes.
- Gravitational wave sources and detection methods.
- What to learn with gravitational waves.

2 Black holes

In 1784, John Mitchell, using newtonian gravity, realized that "dark stars" could exist: stars so compact that light could not escape and would remain black or dark for external observes. He anticipated general relativity ideas, introducing the Schwarzschild radius before it was known:

$$R_S = \frac{2G_N M}{c^2} = \frac{2G_N M_\odot}{c^2}\left(\frac{M}{M_\odot}\right) \simeq 2.95\ km\left(\frac{M}{M_\odot}\right) \tag{2}$$

using the escape velocity of a body of mass M, and the speed of light (nowadays fixed to the value $c = 299792458\ m/s$). $M_\odot$ is the solar mass $M_\odot = 1.989 \cdot 10^{30} kg$. Furthermore, such an object (dark star, black hole) should have a surface gravity

$$g_S = \frac{GM}{R_S^2} = \frac{c^4}{4G_N M} = \left(\frac{c^4}{4G_N M_\odot}\right)\left(\frac{M}{M_\odot}\right) \simeq 1.52 \cdot 10^{13}\ \left(\frac{M_\odot}{M}\right)\ m/s^2 \tag{3}$$

In our galaxy, the Milky Way, we have a supermassive black hole. At the heart of SgA*, near the centre of the galaxy, hides an compact object with

1. $M \approx 4 \cdot 10^6 M_\odot$.
2. $R \approx R_S \simeq R$(Pluto orbit).
3. $L \leq L_\odot$ (it is "dark").

Remarkly, black holes are objects to study "strong gravity". For comparison, the potential energy of gravity $V_g = GM/r$ is such as:

- $\dfrac{V_g}{c^2} \sim 10^{-9}$ for the Earth.
- $\dfrac{V_g}{c^2} \sim 10^{-6}$ for our sun.
- $\dfrac{V_g}{c^2} \sim 1$ for a black hole.

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3 GW sources and detection methods

There are 4 main source types of GW:

- Binary systems (sometimes referred as coalescent binary mergers, CBM).
- Continuous gravitational waves (for instance, we expect them from pulsars).
- Burst sources (e.g., those from cosmic strings, defects, supernovae, phase transitions,...).
- Stochastic gravitational waves (from cosmological non-solved sources).

Binary systems can coalesce emitting gravitational waves:

- BH-BH (black hole-black hole).
- BH-NS (black hole-neutron star).
- NS-NS (neutron star-neutron star).
- BH-WD (black hole-white dwarf).
- WD-NS (white dwarf-neutron star).
- WD-WD (white dwarf-white dwarf).

or any other binary system in general. In the case of neutron stars (NS), we have the following typical parameters:

- $1.25 - 3M_\odot$.
- $R \sim 10 - 20\ km$.
- $1\ ms \leq P_s \leq 100s$.

The strain for a gravitational wave binary can be expressed with the equation

$$h = 10^{-22}\left(\frac{\mathcal{M}}{2.8M_\odot}\right)^{5/3}\left(\frac{0.01\ s}{P_{orb}}\right)^{2/3}\left(\frac{3\cdot 10^{24}\ m}{r}\right) \tag{4}$$

There are big main detection methods and alternative methods (astrometry, resonant cavities, satellite timing,...) for gravitational wave searches. A summary can be found in the next table 1.

4 What to expect from GW

Open questions in astrophysics, cosmology or even particle physics will be hitting with gravitational wave data and we could learn about:

- Dark Matter and Dark Energy.
- Dense matter and neutron star substructure (state equation of matter inside neutron stars).
- Early Universe Physics or even pre-Big Bang physics.
- Fundamental and strong gravity: modified gravity, polarization modes, speed and waveforms,...
- The $M - \sigma$ relation.
- Phase transitions for pre-CMB times and after that.
- Things we can not expect...

Method	Frequency (Hz)	Sources	Detection time
Ground based	10-2000	Binary systems	2016-...
Space based	$10^{-4} - 10^{-2}$	Supermassive BH ($10^6 - 10^7 M_\odot$)	$\sim 2030 - 2040$
Pulsar timing	$10^{-9} - 10^{-6}$	Big supermassive BH ($10^8 - 10^{10} M_\odot$)	$\sim 2020's$
CMB polarization	$10^{-19} - 10^{-16}$	Early Universe Phase transitions Cosmic strings Defects ...	$2020 - 2030's$
Alternative methods	Gaps between the big 4	Astrophysical sources Extreme physics Cosmic strings Exoplanets, SN, r-modes ...	$>$ 2030-...

Table 1: A summary of GW detection methods and sources.

A Thermodynamics and Quantum black holes

Classical Black Holes obey a sort of thermodynamical laws, and quantum mechanics and statistical mechanics suggest black holes have microstates and are quantum objects. For instance, the 4 laws of BH thermodynamics state

1. Black holes in equilibrium have a temperature.
2. In equilibrium, BH have constant surface gravity.
3. The entropy of a black hole scales with the area and it can not decrease.
4. Black hole entropy can not be null.

Pioneer works by Bekenstein and Hawking showed that even static non-rotatory black holes have quantum non null temperature, entropy or pressure:

$$T_{BH} = \frac{\hbar c^3}{8\pi G_N M k_B} \tag{5}$$

$$S = \frac{k_B c^3}{4G\hbar} A_S \tag{6}$$

The quantum pressure is something more curious and not well-developed yet in the literature. For the classical gravity pressure on the event horizon we should have

$$P = \frac{F_g}{A_S} = \frac{M a_S}{A_S} = \frac{c^8}{64\pi G_N^3 M^2} \tag{7}$$

and for the quantum pressure

$$P_q = \frac{E_{BH}}{V_S} = \frac{k_B T_{BH}}{\frac{4\pi R_S^3}{3}} = \frac{3\hbar c^9}{256\pi^2 G_N^4 M^4} \sim 8 \cdot 10^{-42}\ Pa \left(\frac{M_\odot}{M}\right)^4 \tag{8}$$

Imposing $P = P_q$, we get

$$M^2 = \frac{3}{4\pi}\frac{\hbar c}{G_N} \sim \frac{\hbar c}{G_N} = M_P^2 \tag{9}$$

and thus, $M \sim M_P$, as we should expect. In (A)dS spaces we find out the quantum pressure (tension) of vacuum as energy density. In (d)-space-time ($-$ for AdS, $+$ for dS space-time), we have

$$P_\Lambda = \pm \frac{(d-1)(d-2)}{16\pi L_\Lambda^2} = \pm \frac{\Lambda}{8\pi} \tag{10}$$

Black Hole Chemistry can be summarized in the following table:

Classical Thermodynamics	Black Hole (Quantum) Thermodynamics
Enthalpy: $H = E + PV$	Mass-Energy: M
Equilibrium temperature: $\Theta_E = T$	Surface gravity: $g_\Sigma = \frac{\kappa}{2\pi}$
Entropy: $S = k_B \ln N$	Event horizon area: $A_\Sigma = \frac{A}{4}$
Pressure: P	Quantum vacuum pressure: $P_\Lambda = \frac{\Lambda}{8\pi}$
1st Law: $\delta H = T\delta S + P\delta V + \cdots$	1st Law: $\delta M = \frac{\kappa}{8\pi}\delta A + V\delta P + \cdots$
2nd Law: $\delta S \geq 0$	2nd Law: $\delta A \geq 0$

Table 2: Black Hole Chemistry analogy.

Another quantum effect in black holes is the following bound of the ratio of viscosity (shear viscosity) and entropy:

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \tag{11}$$

B Black hole information paradoxes

Black holes at quantum level pose a problem with classical gravity and the theory of General Relativity. Semiclassical arguments by Hawking showed that black holes radiate as black bodies with certain temperature. Also, they should evaporate after a time

$$t_e = \frac{5120\pi G^2 M^3}{\hbar c^4} \simeq 6.62 \cdot 10^{74} \left(\frac{M}{M_\odot}\right)^3 s = 2.10 \cdot 10^{67} \left(\frac{M}{M_\odot}\right)^3 yrs \tag{12}$$

and disappear. But this phenomenon hits a problem with quantum mechanics. Classical Relativity and semiclassical quantum gravity predict BH decay and disappear, but that conflicts with the known theory of Quantum Mechanics, since it currently states quantum objects evolve with the so-called unitary time evolution, conserving information and not loosing it. Therefore, what happens with the final evolution of black holes? Some proposals:

- Information is lost and QM must be changed into a non-unitary theory.
- Information is conserved somehow (in the emitted radiation, in a remmnant, ...).
- A firewall and/or a quantum atmosphere of the BH stores the "lost information".
- Black holes are substituted by other picture or entity at certain point (e.g., the fuzzball picture of string theory and branes).
- Something happens that no one has thought yet.

The Minkovski vacuum can be expressed as the so-called Unruh state

$$|0\rangle_M = \prod_j \sum_{n_j=0}^{\infty} e^{-\pi\omega_j n_j} |n_j, R\rangle \otimes |n_j, L\rangle \tag{13}$$

The final state of the BH after evaporation is currently unknown, and the black hole information paradox remains as a puzzle:

- Quantum Mechanics requires unitarity or unitary evolution.
- Equivalence principle during the evaporation process is a general relativity requirement.
- Effective Quantum Field Theory is imposed in order to keep Hawking effect.

On very general arguments, it is currently known that the above 3 statements can not be compatible each other. Or, at least, no one seems to imagine a way in which the 3 are valid, although there is agreement in that likely it can not be done. That is the essence of the black hole information paradox: you can not keep both general relativity as it stands today and quantum mechanics with the extra of the Hawking effect radiation. Hawking himself created a whole new field proposing that the usual description of quantum mechanics with density matrix were shifted into the so-called superscattering operator in the case of black holes.

One problem of the evaporation process is the destiny of entangled states. Composite systems are the tensor product of sigle states. Suppose 2 systems with Hilbert spaces $\mathcal{H}_A \otimes \mathcal{H}_B$, with dimensions $N_A = \log \dim\mathcal{H}_A$ and $N_B = \log \dim\mathcal{H}_B$, the number of states of such spaces. Then, for maximally entangled states you could write $S_A \approx N_A$, $S_B \approx N_B$, and

$$S_A = -tr_A \rho_A \log \rho_A \rightarrow S_A \approx \frac{\mathbb{I}}{e^{N_A}} \tag{14}$$

$$S_B = -tr_B \rho_B \log \rho_B \rightarrow S_B \approx \frac{\mathbb{I}}{e^{N_B}} \tag{15}$$

and with $N_B \leq N_A$ you could also create the reduced density matrix

$$\rho_A = tr_B \rho_{AB} \tag{16}$$

Entangled states can have maximal entanglement, as the Bell states

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B \right) \tag{17}$$

An interesting property of entangled states is that of *monogamy*: if you have, e.g., 3 systems with Hilbert state $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$, and you have, say, a maximally entangled state within the A and B subsystems, then you can NOT have entanglement between any other (i.e., there is no entanglement between A and C or B and C). The formal reason for this is the known strong subadditivity property of entropies

$$S_{ABC} + S_B \leq S_{AB} + S_{BC} \tag{18}$$

The final state of Quantum BH is not currently fully understood. It's generally believed it will conserve information, but there is no proof of it. Furthermore, it is not clear at all if the whole information can be accessible for all the observers. The solution of this black hole information paradox is a Rosetta Stone for a full quantum theory of gravity and it would shed light about the final fate of black hole singularities like the one in the beginning of the Universe, the one inside black holes (as it is assummed in general relativity) and it would clarify what is the final destiny of the black holes in our universe and the Universe itself as a whole.