Warped Extra Dimensions (WEDs)

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Master on Theoretical Physics. Beyond The Standard Model...

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- Compare WEDs with Large Extra Dimensions (LEDs) scenarios

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- Explain how they can "solve" the hierarchy problem in high energy physics
- Short discussion on the associated phenomenology
- Compare WEDs with Large Extra Dimensions (LEDs) scenarios LEDs do not solve the hierarchy problem indeed, they only shift the problem
- Historical remark: (Warped) Randall-Sundrum models were indeed known before (earlier ideas by Rubakov and Shaposnik) but no popular

The Hierarchy Problem(I): the big issue

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Large Extra Dimensions (LEDs) and gravity in a nutshell

Back to the school: Newton's law for gravity is

$$F = G\frac{Mm}{r^2} = \frac{Mm}{M_{Pl}^2 r^2}$$

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Back to the future(LEDs): Newton's law for gravity in D=4+n dimensions becomes

$$F_D^{Newton} = G_N^{(D=4+n)}\frac{Mm}{r^{2+n}} = \frac{Mm}{M_{Pl(D=4+n)}^{2+n} r^{2+n}}$$

The Hierarchy Problem (II): the LEDs solution

If we live in a 3+1 dimensional world, then we calculate the Gravitational Field via Gauss' law for gravity: $\mathbf{g}(\mathbf{r}) = -Gm\frac{\mathbf{e_r}}{r^2}$ which is simply the Newton's law of gravitation.

The Hierarchy Problem (II): the LEDs solution

Note that

Newton's constant G can be rewritten in terms of the Planck Mass as before :

$$G_N = \frac{1}{M_{Pl}^2}$$

The Hierarchy Problem (II): the LEDs solution

If we extend this idea to δ extra dimensions, then we get: : $\mathbf{g}(\mathbf{r}) = -m\frac{\mathbf{e_r}}{M_{Pl+3+1+\delta}^{2+\delta} r^{2+\delta}}$ where $M_{Pl+3+1+\delta}$ is the $3+1+\delta$ dimensional mass.

The Hierarchy Problem (II): the LEDs solution(2)

Assuming that these extra dimensions are the same size as the normal 3+1 dimensions. Let us say that the extra dimensions are of size n, smaller than normal dimensions. If we let $r << n$, then we get the second relation. However, if we let $r >> n$, then we get our usual Newton's law. When $r >> n$, the flux in the extra dimensions becomes a constant, because there is no extra room for gravitational flux to flow through.

The Hierarchy Problem (III): the LED result

Thus the flux will be proportional to n^δ because this is the flux in the extra dimensions. The formula reads :

$$\mathbf{g}(\mathbf{r}) = -m\frac{\mathbf{e_r}}{M_{Pl+3+1+\delta}^{2+\delta} r^2 n^\delta} \rightarrow -m\frac{\mathbf{e_r}}{M_{Pl}^2 r^2} = -m\frac{\mathbf{e_r}}{M_{Pl+3+1+\delta}^{2+\delta} r^2 n^\delta}$$

which gives:

$$\frac{1}{M_{Pl}^2 r^2} = \frac{1}{M_{Pl+3+1+\delta}^{2+\delta} r^2 n^\delta} \Rightarrow \boxed{M_{Pl}^2 = M_{Pl+3+1+\delta}^{2+\delta} n^\delta}$$

The Hierarchy Problem (III): the LED result

Thus the fundamental Planck Mass (the extra dimensional one) could actually be small, meaning that gravity is actually strong, but this must be compensated by the number of the extra dimensions and their size. Physically, this means that gravity is weak because there is a loss of flux, it is diluted, to the extra dimensions.

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However: the hierarchy problem persists since the extradimensional volume is free. Instead of answering why are the two scales so different, we may ask why Nature selects certain volume!

Warped Extra Dimensions/Geometries (I): the RS(I) main idea?

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Geometry!!!! Not (Warp)speed!!!!!!

Warped Extra Dimensions/Geometries (I): the RS(I) stuff

- Suppose a 5d metric (AdS-like)
  $ds^2 = g_{MN} dx^M dx^N = e^{-A(y)} dx^\mu dx^\nu - dy^2$

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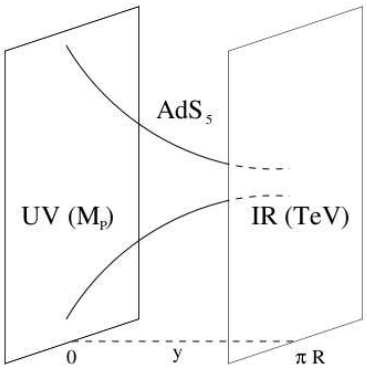
- 

Warped Extra Dimensions/Geometries (II): RS(I) and the hierarchy problem

The 5d Einstein field dimensional field equations $G_{MN} = \kappa^2 T_{MN}$ for the RS metric can be derived from the action

$$S = -\int dx^5 \sqrt{g} M_5^3 R + \Lambda \tag{1}$$

and where $\kappa^2 = 1/M_5^3$ is the 5d gravitational Newton's constant.

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We can also set, without sake of generality, up to a numerical positive constant, $-\Lambda = \kappa^2 M_5^3$, and solving de field equation we get $A(y) = \kappa|y|$ $ds^2 = e^{-\kappa|y|} dx^\mu dx^\nu - dy^2$

Warped Extra Dimensions/Geometries (III): the RS(I) final result and the the RS(II) model

Finally, integrating over the 5-coordinate we would get for the effective 4d world (brane) Planck scale

$$\boxed{M^2_{4d,Planck} = \frac{M^3_5}{\kappa}(1 - e^{-\pi\kappa R})}$$

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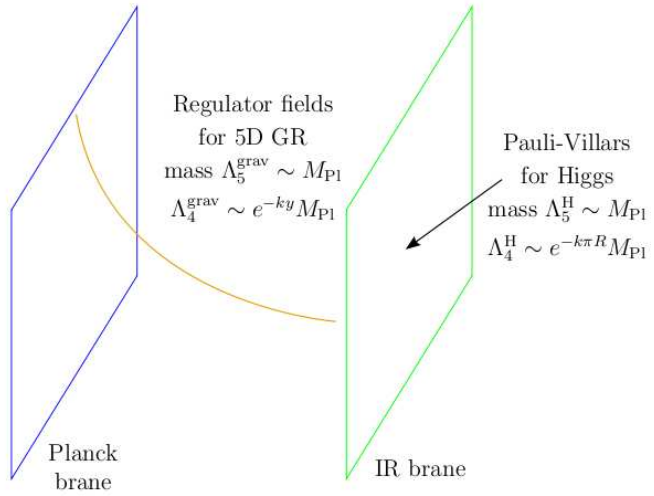
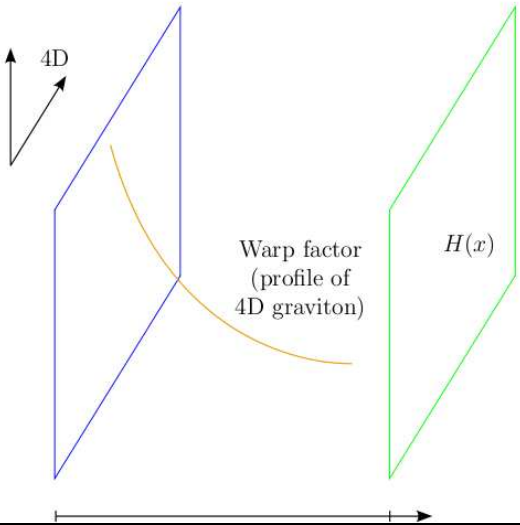
This can help to solve the hierarchy problem from a novel point just setting the cosmological constant parameter. Gravity is weak in our brane, but in 5d gravity is strong!

Warped Extra Dimensions/Geometries (IV): some phenomenological aspects



$$k \lesssim M_5 \lesssim M_{4\,Pl} \sim 10^{18}\text{GeV}$$



Warped Extra Dimensions/Geometries (IV): some phenomenological aspects(2)

KK mode configuration $\quad f^{(n)}(\phi) = e^{kr_c|\phi|}\psi^{(n)}$

$f^{(0)}(\phi) \sim e^{-kr_c|\phi|}$ **localize around hidden brane**

$f^{(n)}(\phi) \sim e^{kr_c|\phi|}\, J_2(m_{(n)}/\kappa\, e^{kr_c|\phi|})$ **localize around visible brane**

$M \sim M_4 \gg M_W$

graviton KK mode mass $m_{KK}^{(n)} \sim x_n \kappa e^{-kr_c\pi} \sim x_n\, \mathcal{O}(M_W)$

$J_1(x_n) = 0 \quad x_1 \sim 3.83$

$x_2 \sim 7.02$

$x_3 \sim 10.17$

$x_4 \sim 13.32$

Collider physics

KK graviton resonance production, if $\sqrt{s} > m_{KK}^{(1)}$

$$\mathcal{L}_{int}^{(1)} = -\frac{1}{\bar{M}_4 e^{-kr_c\pi}} G_{\mu\nu}^{(1)} T^{\mu\nu}$$

$$\bar{M}_4 e^{-kr_c\pi} \sim \frac{m_{KK}^{(1)}}{3.83}\frac{\bar{M}_4}{k}$$

$m_{KK}^{(n)} = x_n k e^{-kr_c\pi}$

$J_1(x_n) = 0$

Model parameters → $\frac{k}{\bar{M}_4}$, $m_{KK}^{(1)}$

(Warped) CONCLUSIONS

In warped extra dimension scenario

$$V_5 = 2r_c \int_0^{\pi} d\phi\, e^{-2kr_c|y|} = \frac{1}{k}\left(1 - e^{-2kr_c|y|}\right)$$

$$\rightarrow \quad M_4^2 = \frac{M^3}{k}\left(1 - e^{-2kr_c\pi}\right)$$

Effective volume is finite even if $r_c \to \infty$

implies → Alternative compactification scenario

New property → Geometry

Geometrical meaning → why EW scale is so small?

$$M_W \ll M_4 \sim 10^{19}\text{GeV}$$

Large extra-dimension scenario:

$$M_{4+\delta} \sim \mathcal{O}(\text{TeV}) \sim M_W$$

$$M_{4+\delta} = \left(\frac{M_4}{V_\delta}\right)^{\frac{1}{2+\delta}}$$ ← dilution by large extra-dimensional volume

Warped extra-dimension scenario:

$M_4 \sim M_5 \sim 10\kappa$ ← Mild hierarchy

$$M_W = M_4 \times e^{-\pi\kappa r_0}$$ ← suppression by ``warp'' factor

(Warped) CONCLUSIONS

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implies → **Alternative compactification scenario**

RS models are also interesting due to the AdS/CFT correspondence, stability, continuous KK spectrum vs. discrete (LEDs) but we do not know if Nature obeys it (warped geometry). In fact, we live in a dS, not AdS Universe, it seems to be...Very hard work about these models and their collider signals done and to be done...

(Warped) CONCLUSIONS

THANK YOU!