Homework: Solar Sails

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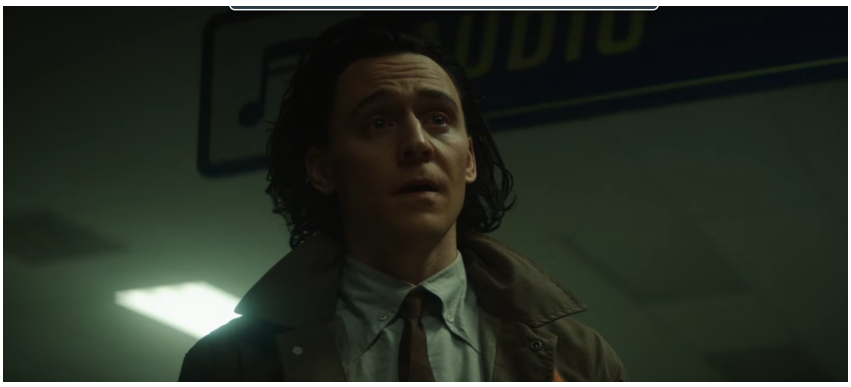
Abstract

Homework on starsails and the Starshot project.



1 Introduction: Sailing with Light

Photons have a momentum that depends on the energy of the light. Since $E_f = m_f c^2$, then $p_f = E_f/c = mc$. This momentum can be transferred to bodies hit or impacted by the light.

The idea of using radiation pressure from sunlight or gigantic laser systems to propel space probes has been around for a long time. If a very thin, highly reflective foil is stretched across huge sails, photons will bounce off the foil and transfer their momentum to the sails. This worksheet presents a concrete example of an application of this concept. The momentum of photons can also be used on microscopic scales to move small objects. Today, optical tweezers, the invention of which was awarded the Nobel Prize in Physics in 2018, play an important role in medical and biological research.

2 Solar sails and radiation pressure: Journey to Proxima Centauri

In 2016, an Earth-like exoplanet was discovered orbiting the habitable zone of the nearby star Proxima Centauri b. Following this discovery, an extremely bold and visionary idea was published explaining how this planet, 4.2 light years away, could be reached with a space probe within a travel time of 20 years. To propel the space probe, an extremely light but stable light sail was suggested. Although the whole thought experiment may seem somewhat utopian, it is quite interesting to take a closer look at the physics behind the project.

2.1 Exercise 1

Learn more about the visionary project called "Breakthrough Starshot", for example at the following link: `https://blogs.scientificamerican.com/life-unbounded/can-starshot-work/`

2.2 Exercise 2

Next, we will calculate how much momentum the photons can transfer to the light sails. Suppose the spacecraft is moving away from us at constant speed with momentum p_S. The sails are hit by photons with total momentum P_f. Since the mass of the spacecraft is much, much greater than the mass of the photons, the photons are reflected as if they were a wall, so they keep their momentum but change direction. This elastic collision transfers a momentum Δp to the sail.

To calculate the momentum increment, we can use the following equation:

$$P_S + P_f = (P_S + \Delta p) - P_f \tag{1}$$

a) Explain this equation as a result of a conservation law.

b) Deduce that the momentum transferred to the sail by the photons is $\Delta p = 2P_f$.

c) Light has energy E_L and momentum $P_f = E_L/c$. Using $\Delta = m\Delta v$, show that the velocity of the spacecraft of mass m_S increases by an amount $\Delta v = \dfrac{2E_L}{m_S c}$ when it is struck by the light.

d) The Starshot project proposes using an array of laser cannons to generate enormous amounts of photon pressure on the sail. The proposed energy for the laser cannons is 100 GW. Compare that energy to that of current lasers and the hypothetical laser on the Death Star in Star Wars.

e) Determine the energy emitted per second by these cannons, and the speed increase per second (acceleration) associated with the sails of the spaceships, if we make the unrealistic (or idealistic) hypothesis that the total mass of the ship is 1.33 grams, that all photons impact the solar sails, and that all photons are totally reflected.

f) Calculate the speed increase after 120 seconds of exposure to the laser cannons and the speed reached by the microship compared to the speed of light. Would we expect relativistic effects at these

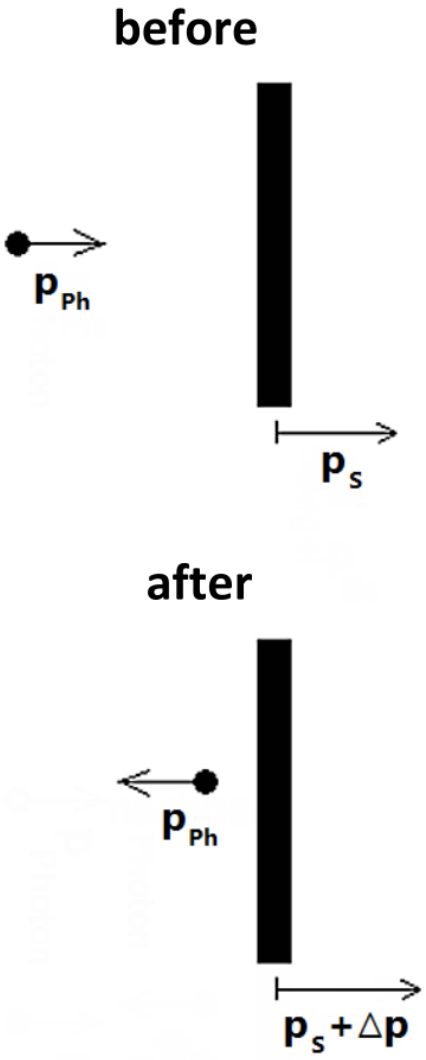


Figure 1: Photons impacting against a solar sail. Abstract picture.

speeds?

g) Calculate the travel time to Proxima Centauri by the ships from the point of view of Earth and the ships.

h) Research optical tweezers and their applications.

i) Research the applications of lasers in Physics.