Introduction to String Theory: final assignments

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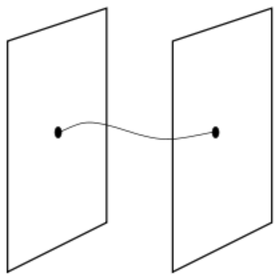
Prof.: *Ángel Uranga*, a really friendly anhilator and Dp-braned stringy man. Thanks for your instructive course and notes!

✍Exercise 7: D-branes and the Higgs (Solution)

"Deep brains thinking with screens about the Higgs...Does it exist?Do they exist?Stay tuned..."



Following the classnotes and the lecture notes, we consider open strings type IIB in D=10. A configuration of D3branes along x^9, situated in $a_1, a_2, \ldots, a_N$. A generic D3-brane is then denoted by x^a or x^b, according to its position. We proceed to the solution of the problem in general, i.e. for any odd p, and then we will comment the particular result with $p = 3$ when neccesary:

a) The N^2 open strings sectors, labelled by the latin letters a and b that corresponds to open strings starting at the a-Dbrane and ending on a b-Dbrane, have NN boundary conditions for 2d bosons $X^\mu(\sigma, \tau)$ and DD boundary conditions for the fermionic $X^i(\sigma, \tau)$. Remember we set coordinates x^μ,$\mu = 0, \ldots, p$ for the directions on the D-brane and x^i, $i = p + 1, ..., 9$, with $9 - p$ transverse degrees of freedom or directions "perpendicular"/ortogonal to the brane. The D-brane boundary conditions (BBC) read

$$\partial_\sigma X^\mu(\sigma, \tau)|_{\sigma=0,l} = 0 \tag{1}$$

$$X^i(\sigma, \tau)|_{\sigma=0} = x_a^i; \qquad X^i(\sigma, \tau)|_{\sigma=\tau} = x_b^i \tag{2}$$

The mode expansion for the string attached to the D-branes are

$$X^i(\sigma, \tau) = x_a^i + \frac{x_b^i - x_a^i}{l}\sigma + i\sqrt{\frac{\alpha'}{2}} \sum_n \frac{\alpha_n^i}{n} e^{-\pi i n(\sigma+\tau)/l} + \sum_n \frac{\tilde{\alpha}_n^i}{\tilde{n}} e^{-\pi i n(\sigma-\tau)/l} \tag{3}$$

and it can be understood as the decomposition of the attached D-brane in four terms: the D-brane position term, the center of mass term, the oscillating left movers term and the oscillating right movers term. Moreover, we remember too the difference between the BBC in the NN and DD cases:

$$\alpha_n^\mu = \tilde{\alpha}_n^\mu, \qquad \text{for} \quad n = m \in \mathbb{Z}, \quad x^\mu, p^\mu \qquad \text{allowed}$$

NN-type BBC and

$$\alpha_n^i = -\tilde{\alpha}_n^i, \qquad \text{for} \quad n = m \in \mathbb{Z} \quad x^i \quad \text{fixed}, p^i = 0$$

in the NN-type BBC case. The spectrum is very similar to that of pure open string, with an additional piece coming from the winding and the position of the D-branes. The mass formula will be:

$$\boxed{M^2 = \left(\sum_{i=p+1}^{9} \frac{x_b^i - x_a^i}{2\pi\alpha'} \right)^2 + \frac{1}{\alpha'} (N_B + N_F + E_0)} \tag{4}$$

where $E_0 = -1/2, 0$ if we are in the NS or the R sector, respectively. Here we could put the explicit $p = 3$ requirement. For the massless case, $M^2 = 0$, the NS and R D-brane states of the spectrum define a $U(N)$ field theory since they are, for NS-BBC, fermions $\psi^{ab}_{-1/2}|0\rangle$ plus six scalars ϕ^i and for R-BBC 8_c^{ab} and extra 4+4 fermionic Weyl spinors λ_+, λ_-, all these fields transforming in the adjoint representation of the little group $SO(p-1)$. We have then N^2 gauge bosons, $(9-p)$, in our case $9-3=6$, times real scalars and $2^{9-p/2}$ times N^2 chiral fermions in $p+1$ dimensions (field theory). In summary, the massless excitations of N coincident branes are a U(N) gauge field $(A_c)^a_b$ together with scalars $(\phi^M)^a_b$ which transform in the adjoint representation of the U(N) gauge group. The fastest way to see that N coincident branes give rise to a U(N) gauge symmetry, and its breakdown to N times U(1) symmetry, is to realize that the end point of the string is charged under the U(1) gauge field that inhabits the brane it's ending on. Let's put the simplest example and then prove the result by induction.Suppose that we have two branes. The diagonal components $(A_a)^{11}$ and $(A_a)^{22}$ arise from strings which begin and end on the same brane. Each is a U(1) gauge field. What happens with the off-diagonal terms $(A_a)^{12}$ and $(A_a)^{21}$? These come from strings stretched between the two branes. They are again massless gauge bosons, but they are charged under the two original U(1) symmetries; they carry charge $+1, -1$ and $-1, +1$ respectively. But this is precisely the structure of a U(2) gauge theory, with the offdiagonal terms playing a role similar to W-bosons in the Standard Model. Repeating the argument for N branes, we are left with the U(N) gauge theory, as we wanted to prove.

b) We can see than whenever the brane positions coincide, $ab = aa$, we obtain massless states whatever are the positions of them, and that we have an $U(1)^N$ symmetry. Why? It is easy. The spectrum formula can be imagined as a matrix M_{ab}, $N \times N$. When the branes coincide, we have a diagonal matrix proportional to the unit. So then, we have a global phase symmetry in each entry on the diagonal. The off-diagonal terms are precisely the massive states of the spectrum when the branes are separated, as it can be trivially seen from the mass spectrum above, since the will be non-zero and the boson and fermion contributions can not erase their contribution. The mass formula after the separation, neglecting the boson and fermion numbers, and the vacuum energy too, is simply

$$\boxed{M^2 = T^2(x_2 - x_1)^2} \tag{5}$$

where $T = 1/2\pi\alpha'$ is the string tension. More generally, we obtain:

$$\boxed{M^2_{ab} = T^2(x_b - x_a)^2} \tag{6}$$

c) The low energy action for N coincident Dp-branes is:

$$S = -(2\pi\alpha')^2 \int d^{p+1}\xi Tr \left(\frac{1}{4} F_{ab}F^{ab} + \frac{1}{2} D_a\phi^M D^a\phi^N - \frac{1}{4} \sum_{M \neq N} \left[\phi^M, \phi^N\right] \right) \tag{7}$$

where $F_{ab} = \partial_a A_b - \partial_b A_a + i\,[A_a, A_b]$. When we turn on one of the scalars, we set,e.g., for two branes (generalization is trivial by induction)

$$\phi = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix} \tag{8}$$

The mass of the gauge field that comes from the above Dp-brane action follows from the covariante derivative term, expanding the gauge field, we obtain the matrix

$$A_c = (A_c)^a_b \begin{pmatrix} A_c^{11} & W_c \\ W_c^{\dagger} & A_c^{12} \end{pmatrix} \tag{9}$$

Computing the term coming from the commutators

$$\frac{1}{2} Tr\,[A_c, \phi]^2 = -(\phi_2 - \phi_1)^2 |W_c|^2$$

so we have the mass of the W-boson field with the formula

$$M_W^2 = (\phi_2 - \phi_1)^2 = T^2 |x_2 - x_1|^2 \tag{10}$$

where T is the tension of the string, $T = 1/2\pi\alpha'$, and we could see the match with the string mass spectrum with D-branes we computed in a).

Conclusion: D-branes seem to provide a geometric framework or interpretation for the mysterious Higgs mechanism we have invented to give masses in the Standard Model. Of course, Higgs particles have not been found yet, but it is useful to ask where the Higgs mechanism comes from. But then, the question would be...Where the D-branes come from? Of course, they are nonperturbative states in string theory but they are not completely understood yet.

d) In the string set-up, the residual or remmant permutation symmetry S_N says to us that D-branes are indistinguible objects (we should not say particles, since D-branes are, in general, extended objects excepting the p=0 case). In the literature, the subgroup of U(N) which leaves invariant the permutation group is sometimes called Weyl group.

✍Exercise 8: Couplings, scales and extra dimensions in heterotic and type II superstrings (Solution)

"String theory is not only a theory of strings, branes change and "save" the world...Forever?"

The exercise has two parts, each of them with four subparts a, b, c, and d. We call A and B those parts. And now, we proceed to the solution:

PART A: THE HETEROTIC STRING AND THE OLD GRAVITY/GAUGE UNIFICATION AT STRING SCALE.

A.a. The heterotic string on the ten dimensional manifold $\mathcal{M}_4 \times X_6 = G$ has the following effective action, corresponding to the genus expansion lowest order (g=0) in the gravity and the gauge sector (additional corrections in g_s are neglected):

$$S^{eff}_{D=10} = S_{gravity} + S_{gauge} = \int_G d^{10}x \frac{M_s^8}{g_s^2}\sqrt{-G_{(D=10)}}R_{(D=10)} + \int_M d^{10}x \frac{M_s^6}{g_s^2} F^2_{(D=10)} \tag{11}$$

The dependence on the string coupling is understood as we are considered the lowest order un the genus/Euler characteristic expansion on the worldsheet (spheres). The additional mass terms are neccesary in order to get the correct dimension for the action, i.e., $ET = Action = Energy \times Time$. We can check this as follows:

- The gravitational action (11) has dimensions $TL^9E^8EL^{-1} = \mathbf{ET}$, as required, since in natural units we can measure mass and energy as the same concept (here E, energy) and the inverse length L^{-1} has dimension of E, energy, too. Note that the curvature R has dimension of "force", that is, $EL^{-1} = L^{-2} = E^2$.
- The gauge piece of the action has dimension $TL^9E^6E^2L^{-2} = \mathbf{ET}$. Note that the field strength F has dimension of force, as the gravitational curvature as well[1].

A.b. The 10d effective action can be dimensionally reduced down (a.k.a. "compactified") to four dimensions integrating over the "invisible" or "hidden" dimensions (represented by the compact space X_6):

$$S^{eff}_{D=4} = S_{gravity} + S_{gauge} = \int_G d^4x d^6x \frac{M_s^8}{g_s^2}\sqrt{-G_{(D=10)}}R_{(D=10)} + \int_G d^4x d^6x \sqrt{-G_{(D=10)}}\frac{M_s^6}{g_s^2} F^2_{(D=10)} = \tag{12}$$

$$V_6 \int_M d^4x \frac{M_s^8}{g_s^2}\sqrt{-G_{(D=4)}}R_{(D=4)} + V_6 \int_G d^4x \sqrt{-G_{(D=4)}}\frac{M_s^6}{g_s^2} F^2_{(D=4)} = \tag{13}$$

$$V_6 \frac{M_s^8}{g_s^2} \int_M d^4x \sqrt{-G_{(D=4)}}R_{(D=4)} + V_6 \frac{M_s^6}{g_s^2} \int_G d^4x \sqrt{-G_{(D=4)}} F^2_{(D=4)} \tag{14}$$

A.c. Identifying the coefficients in the 4d dimensionally reduced effective action with the Planck Mass (Newton's gravitational constant) and the Yang-Mills coupling constant, we obtain the important relations:

$$\boxed{M^2_{P,D=4} = V_6 \frac{M_s^8}{g_s^2}}$$

$$\boxed{\frac{1}{g^2_{YM}} = V_6 \frac{M_s^6}{g_s^2}}$$

[1] At least on these dimensional and conceptual background: *Force is curvature!* However, they are two different forms of curvature: one is the own curvature of spacetime, and the other is a 2-form curvature defined on a principal bundle.

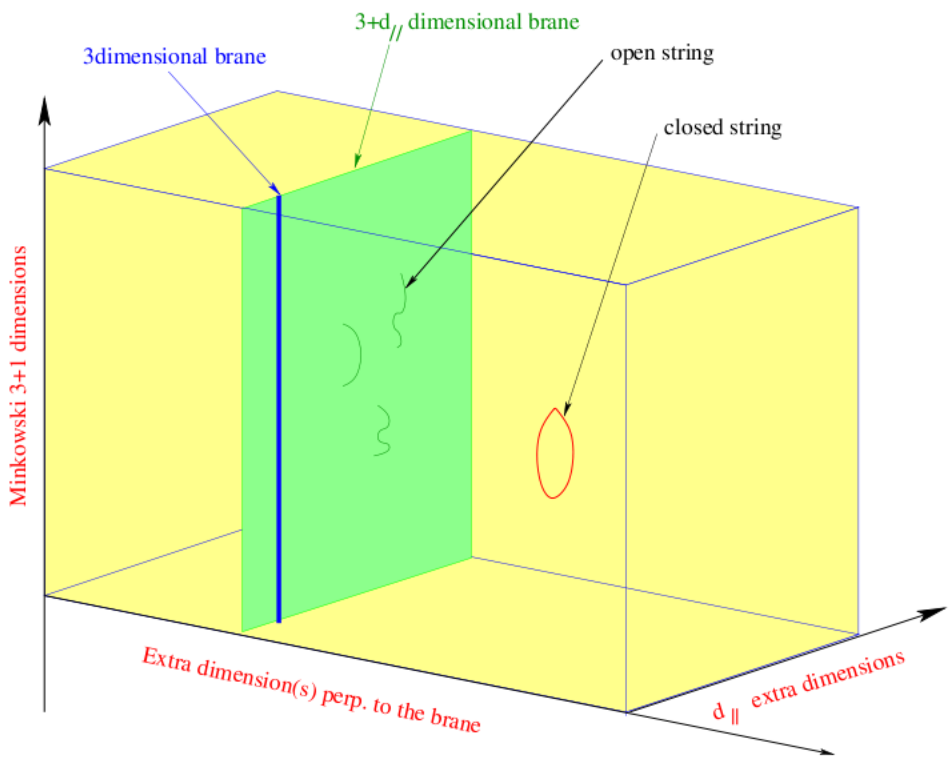
If we insert the value of the hexadimensional volume from the last equation into the former, we can see that

$$\boxed{M_{P,D=4}^2 = \frac{M_s^2}{g_{YM}^2}}$$

and then if we impose by hand that $g_{YM}^2 \sim 1$, we get $M_{P,D=4}^2 \sim M_s^2$, i.e., $M_{P,D=4} \sim M_s$. This can be understood as a gravity/gauge unification at Planck/String scale, and it can be called "*the old high-energy unification*".

A.d.. On the size of the internal volume V_6. The present experimental bounds on extra dimensions for KK replica of gravitons, gluons and other gauge fields has been studied in different experiments and searches for signals in colliders (specially Hera, LEP and Tevatron, the D0 collaboration specially) and they are going to continue in the near future, included in LHC. The present bound on the KK gluon replica, searching a dielectron (electron-positron decay of gluonic KK) is about $E > 1.1TeV$. The precision measures of electroweak observables can increase this limit up to about $3 - 5TeV$, so the limit is precisely in the zone where LHC is going to make collisions in the next two years[2].

PART B: TYPE II STRINGS AND A BRAVE NEW BRANEWORLD WITH Dp-BRANES.



B.a. Dp-branes wrapped along $p-3$ directions and filling the 4d Minkovski spacetime completely. Considering a toroidal compactication for simplicity and denoting by $V_{\|}$ the volume around the internal direction

[2] I am wondering if there are more extradimensional reasons to search for interesting and unknown physics there...

on the D-brane, $V_\perp$ the volume tranverse to the D-brane and $V_6 = V_\perp V_\parallel$ the total internal volume of the D-brane. The effective action for gravitational and gauge interactions *before* the compactification is:

$$S^{eff}_{D=10} = S_{gravity} + S_{gauge,Dp-brane} = \int_G d^{10}x \frac{M_s^8}{g_s^2}\sqrt{-G_{(D=10)}}R_{(D=10)} + \int_W d^{p+1}x \frac{M_s^{p-3}}{g_s} F^2_{(D=p+1=10)} \quad (15)$$

The gravitational action is the same as the heterotic gravitational action, but we have a new gauge piece, due to the Dp-brane. Using the same analysis than the heterotic case, we can see that $TE^{-p}E^{p-3}E^2L^{-2} = \mathbf{ET}$, as we desired to match action dimensions. By the other hand, the explanation on the power of the string coupling constant is the following: the Euler characteristic for an arbitrary worldsheet, the weight in the string coupling constant expansion, in this case is $\chi = 2-2g-b-c$. For a Dp-brane, a kind of "wall", we get $g=c=0$ but $b=1$, and then the power of g_s is one.

B.b. The 10d effective action can be dimensionally reduced down (a.k.a. "compactified") to four dimensions integrating over the "invisible" or "hidden" dimensions (represented by the compact space X_6 and the transverse directions to the Dp-brane, $V_\perp$):

$$S^{eff}_{D=4} = S_{gravity} + S_{gauge} = \int_G d^4x d^6x \frac{M_s^8}{g_s^2}\sqrt{-G_{(D=10)}}R_{(D=10)} + \int_W d^{p+1}x \sqrt{G_{(D=10)}}\frac{M_s^{p-3}}{g_s} F^2_{(D=p+1=10)} = \quad (16)$$

$$V_6 \int_G d^4x \frac{M_s^8}{g_s^2}\sqrt{-G_{(D=4)}}R_{(D=4)} + \int_W d^4x d^{p-3}\sqrt{-G_{(D=4)}}\frac{M_s^{p-3}}{g_s} F^2_{(D=p+1=10)} = \quad (17)$$

$$V_6 \frac{M_s^8}{g_s^2}\int_G d^4x \sqrt{-G_{(D=4)}}R_{(D=4)} + V_\parallel \frac{M_s^{p-3}}{g_s}\int_{W_4} d^4x \sqrt{-G_{(D=4)}} F^2_{(D=4)} \quad (18)$$

B.c. Identifying the coefficients in the 4d dimensionally reduced effective action with the Planck Mass (Newton's gravitational constant) and the Yang-Mills coupling constant, we obtain the important relations:

$$\boxed{M^2_{P,D=4} = V_6 \frac{M_s^8}{g_s^2}}$$

$$\boxed{\frac{1}{g^2_{YM}} = V_\parallel \frac{M_s^{p-3}}{g_s}}$$

If we insert the value of the hexadimensional volumen as a function of the parallel and transverse volumes of the brane at the last equation, and plugging the result into the former, we can see that

$$\boxed{M^2_{P,D=4} = \frac{M_s^{11-p}}{g_s}\frac{V_\perp}{g^2_{YM}}}$$

And now, setting the string and the gauge coupling to the unit, we discover the important equation

$$\boxed{M^2_{P,D=4} = M_s^{11-p}V_\perp}$$

that is

$$\boxed{M_{P,D=4} = M_s^{\frac{11-p}{2}}\sqrt{V_\perp}}$$

We can express the above results in a more useful and common equation from the literature:

$$\boxed{M_{P,D=4}^2 = M_s^{2+n} V_\perp}$$

or, supposing a toroidal compactification in the tranverse space,

$$\boxed{M_{P,D=4}^2 = M_s^{2+n} R^n}$$

where R is the radius of the torii. We realized that the brane has $9-n$ dimensions, where n is the number of tranverse dimensions, that is, $dimV_\perp = n$, and that the parallel space on the brane has $dimV_\parallel = 6-n$.

The result means that we could get a very low string mass scale, perhaps in the TeV scale, introducing a big enough transverse volume, symple varying R.

B.d. Then, both cases imply the return to the old fashioned scenario whenever we identify the string and the gauge coupling. If we do not that, we are left with an exciting new scenario, depending on the number of extra tranverse dimensions of the D-brane, and that changes the extradimensional phenomenology. A big transverse space could allow us to low the string mass, but no too much since then we would have observe some KK replica or tower of massive states. The present experimental distances probed with Tevatron (waiting soon some results from LHC) are about the fermi, or some order of magnitude lower. From the picture of the braneworld, the parallel brane interacts with us through gauge fields, and as we have no seen KK gluons, for instance, we could claim that at present the experimental bound is about $d_\parallel < 0.1fm$, but, at first, then, we could have, since no experiment has probed that distance, that $d_\parallel \sim 10^{-18}/10^{-20}m$, two or three orders of magnitude. A more precise bounds can be seen from the PDG. By the contrary, on the size of the tranverse space, where gravitons can move, the experimental bound is softer. Searches for modified gravitational law in the form of a Yukawa potential have been done, with negative results till now. A summary of data from 2006 is presented in this table, including bounds using SN1987A and more (we refer to the reader to the particle data booklet o the full particla data review for more updated information):

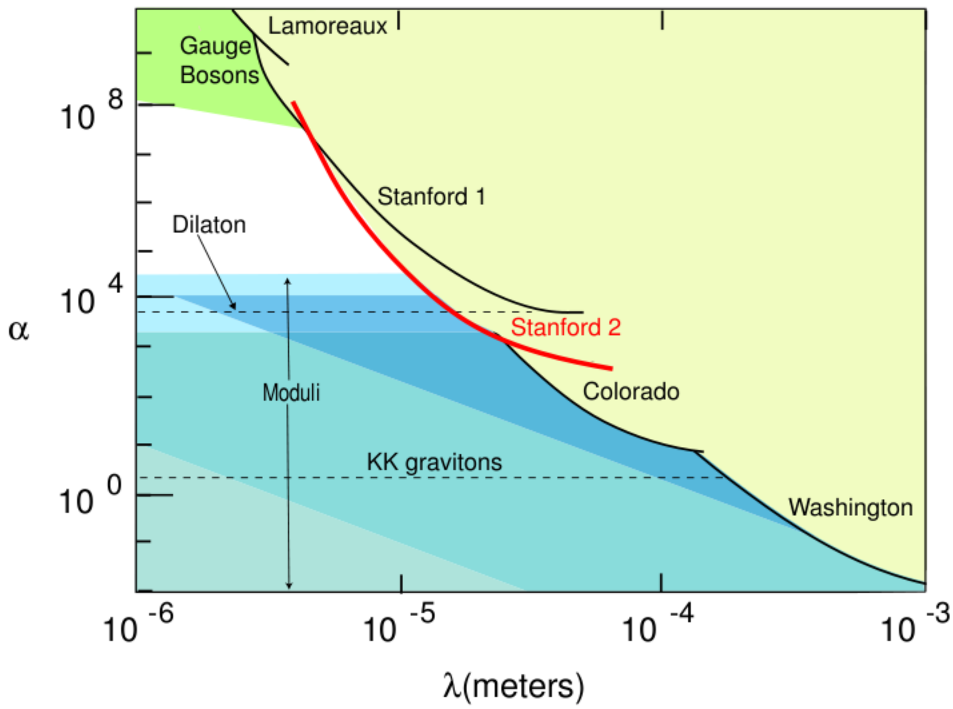
Table 1: Limits on $R_\perp$ in mm.

Experiment	$n=2$	$n=4$	$n=6$
Collider bounds			
LEP 2	5×10^{-1}	2×10^{-8}	7×10^{-11}
Tevatron	5×10^{-1}	10^{-8}	4×10^{-11}
LHC	4×10^{-3}	6×10^{-10}	3×10^{-12}
NLC	10^{-2}	10^{-9}	6×10^{-12}
Present non-collider bounds			
SN1987A	3×10^{-4}	10^{-8}	6×10^{-10}
COMPTEL	5×10^{-5}	-	-

Gravitational Yukawa terms in submilimetric distances have been searched for years. The modified gravitational potential energy reads

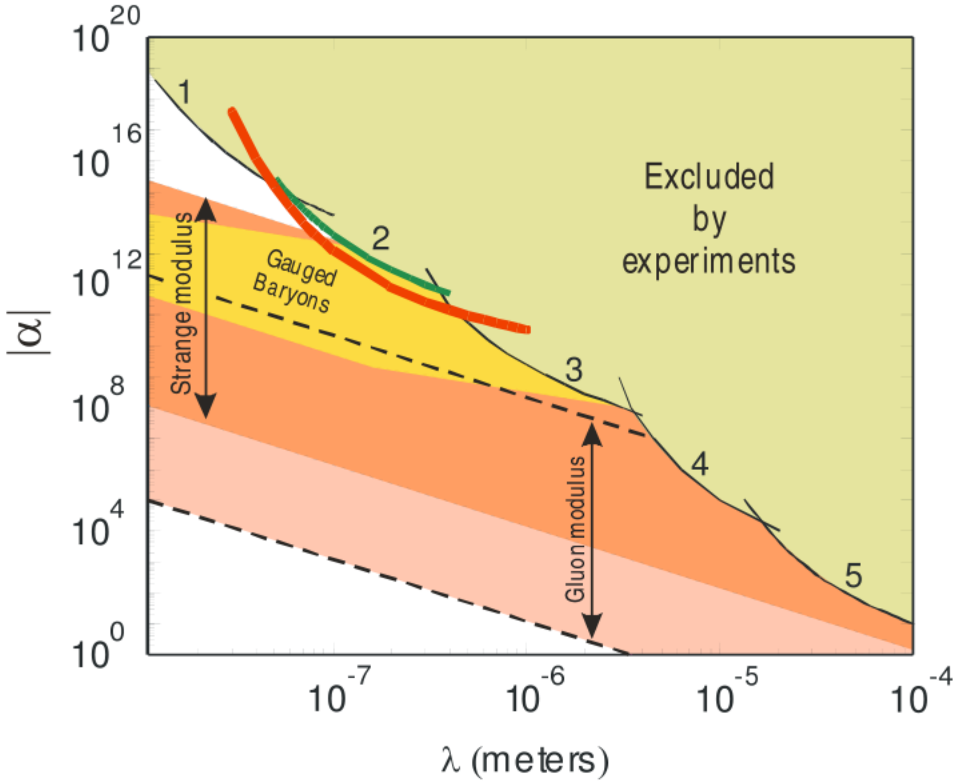
$$U = -G\frac{M_1 M_2}{r}(1-\alpha e^{-r/l}) \tag{19}$$

Bounds on non-Newtonian forces in the range $6-20\mu m$ are

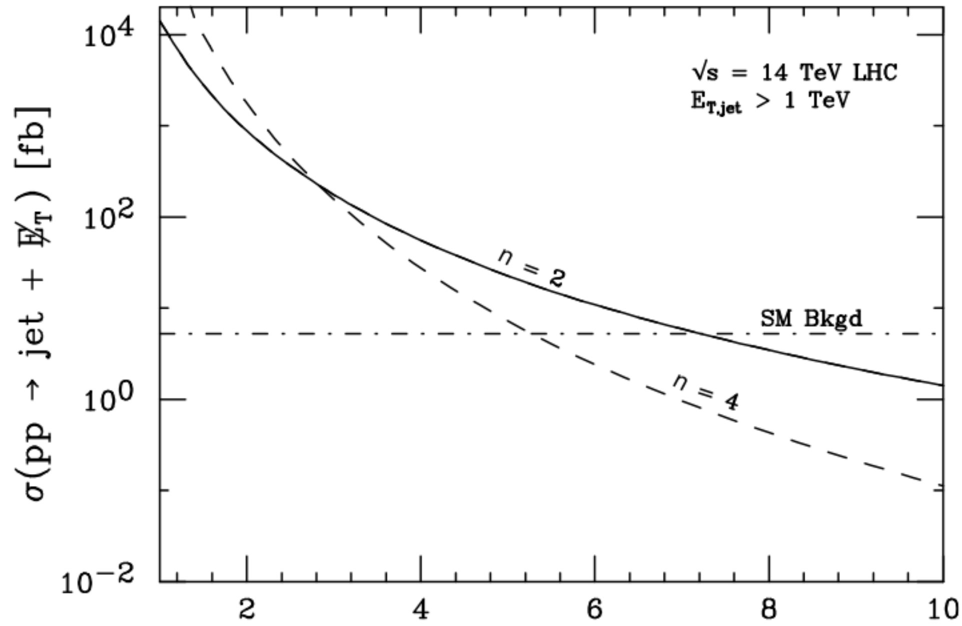


Lamoreaux
Gauge
Bosons
Stanford 1
Dilaton
Stanford 2
Moduli
Colorado
KK gravitons
Washington
10 8
10 4
10 0
α
10 -6
10 -5
10 -4
10 -3
λ(meters)

Bounds on non-Newtonian forces in the range $200nm$ are here



And the final tables and results, a discovery potential analysis, since fuzzy and always "in motion" future. First, the near future and present, the LHC:



and beyond... who knows?

Sensitivities on $R_{\|}^{-1}$ (TeV)			
		Resonances discovery	
Collider	gluons	$W^{\pm}$	$\gamma + Z$
LHC(100 fb^{-1})	5	6	6
Observation of deviations			
Collider	gluons	$W^{\pm}$	$\gamma + Z$
Tevatron (2 fb^{-1})			1.2
Tevatron (20 fb^{-1})	4		1.3
LHC (10 fb^{-1})	15	8.2	6.7
LHC (100 fb^{-1})	20	14	12
LC ($\sqrt{s}$ = 500 GeV, 75 fb^{-1})			8
LC ($\sqrt{s}$ = 1000 GeV, 200 fb^{-1})			13